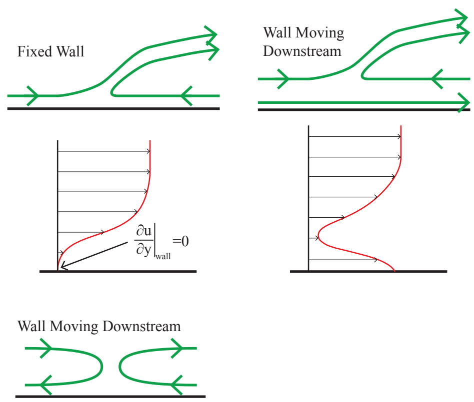
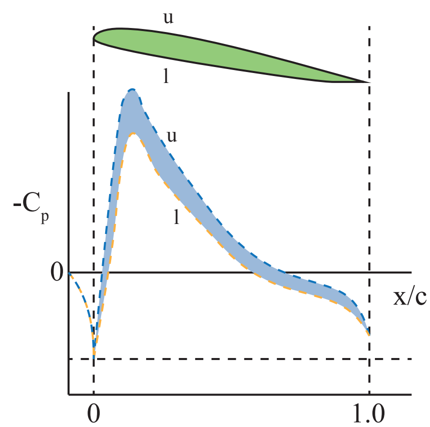
Extrovert E-book Series

Low Speed Aerodynamics

Narayanan M. Komerath





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Disclaimer

Low Speed Aerodynamics

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Chapter 1

Basic Concepts and Results in Aerodynamics

1.1 Force Balance in Flight

- The wings (and the horizontal tails in some cases) support the weight of the aircraft at subsonic speeds.
- The rest of the aircraft just hangs from these "lifting surfaces."
- Of course the wings and tails themselves have weight.
- On most aircraft, the wings contain most of the fuel.

We can use Newton's Laws of Motion to calculate the acceleration of an aircraft, and thus to decide how the forces on the aircraft must be balanced to make it go in a desired direction.

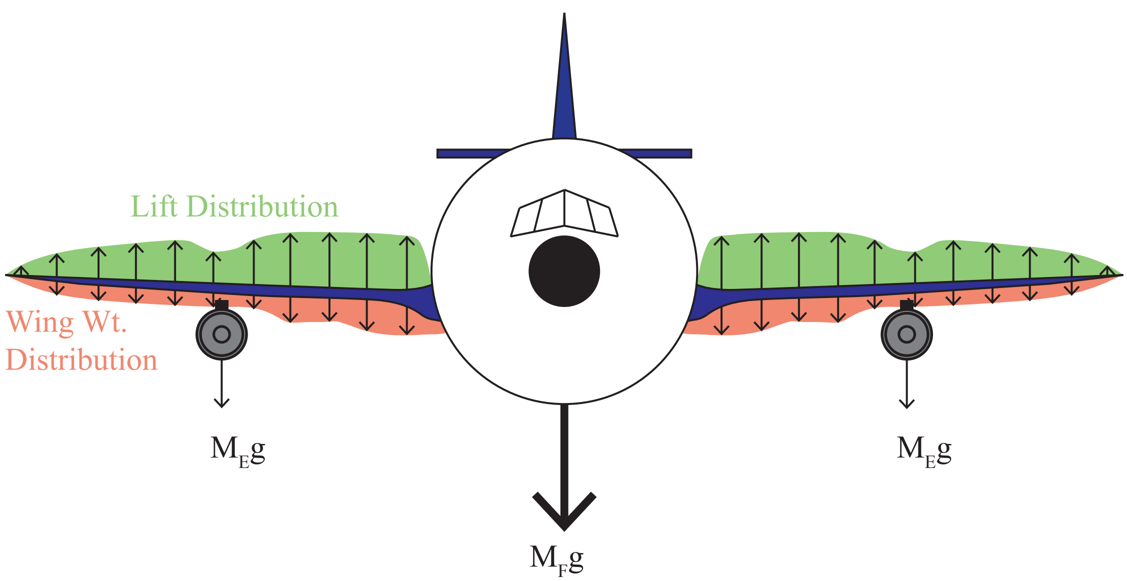


Figure 1.1: Force Balance in Flight

1.2 Straight And Level Steady Flight

In Straight and Level Steady Flight, where all the accelerations are zero, Lift = Weight, and Thrust = Drag.

$$L = W \tag{1.1}$$
$$T = D \tag{1.2}$$

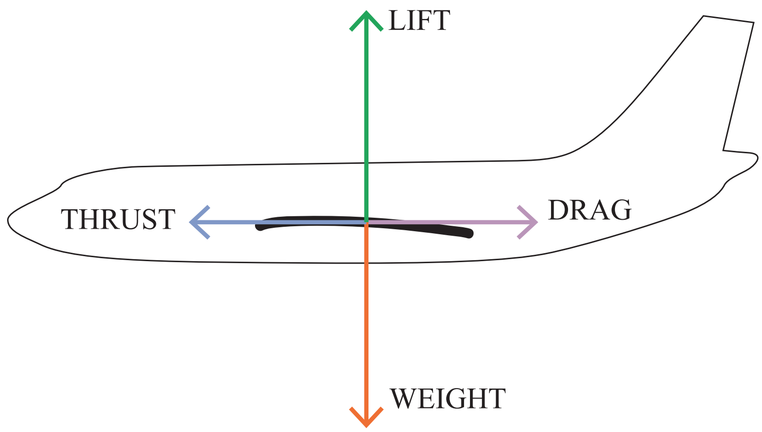


Figure 1.2: Straight and Level Steady Flight

1.3 Basic Concepts and Results in Aerodynamics

Freestream Vector: Velocity of the fluid far ahead of the object in the flow, undisturbed by the presence of the object.

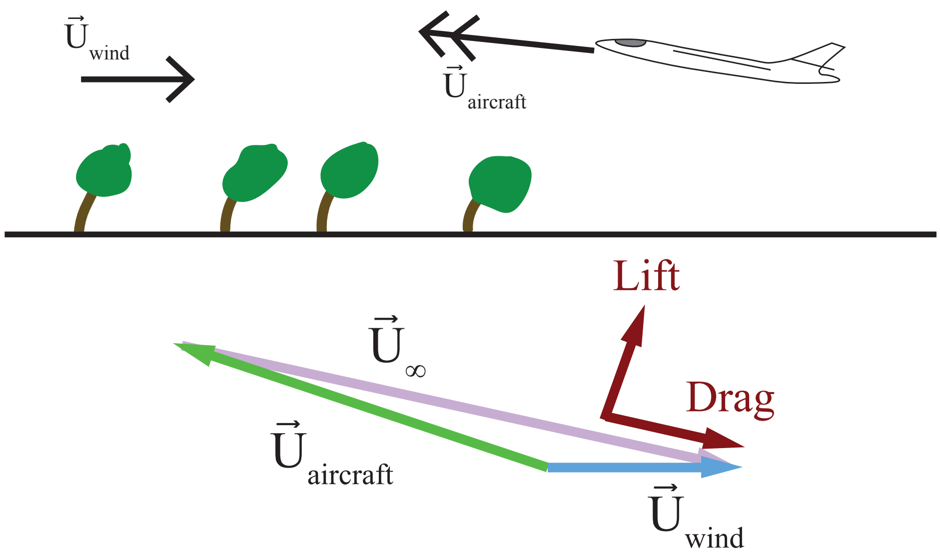
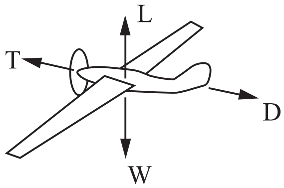


Figure 1.3: Freestream Vector



For straight and level flight, $L = W$ and $T = D$.

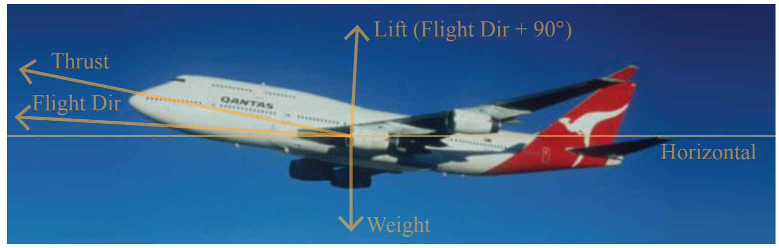


Figure 1.4: Lift, weight, thrust and drag

1.4 Planform Area S

S is the area enclosed by a full-scale drawing of the outline of the wings when viewed from directly above or below.

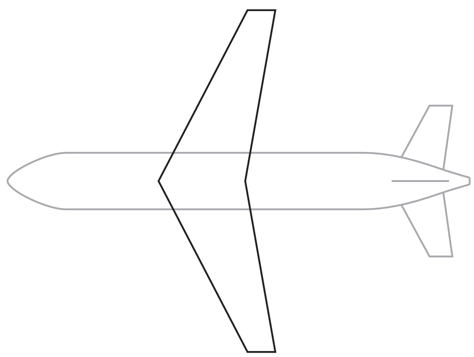


Figure 1.5: Planform Area S

1.5 Dynamic Pressure in low-speed flow

$$q_\infty = \frac{1}{2}\rho U_\infty^2 \tag{1.3}$$

ρ kg/m^3	U m/s	U fps	q N/m^2	q psf
1.2	10	32.8	60	1.253
1.2				
0.36				

$$\begin{aligned} \text{Lift} &= (\text{dynamic pressure}) \times (\text{planform area}) \times (\text{lift coefficient}) && (1.4)\\ L &= q_\infty S C_L && (1.5)\\ L &= \frac{1}{2}\rho U_\infty^2 S C_L && (1.6)\end{aligned}$$

The lift coefficient depends on how the lift was generated. C_L is usually around 0.1 to 1.4.

1.6 Lift Generation

Generation of aerodynamic lift can be explained from Newton's laws of motion. When flowing air is induced to turn, there is a rate of change of momentum along the direction perpendicular to the original direction. The net force which is required to cause this change, causes an equal and opposite reaction on the object which turns the flow. This reaction is the force acting on the object. Most simply there are three different "passive" methods to turn the flow.

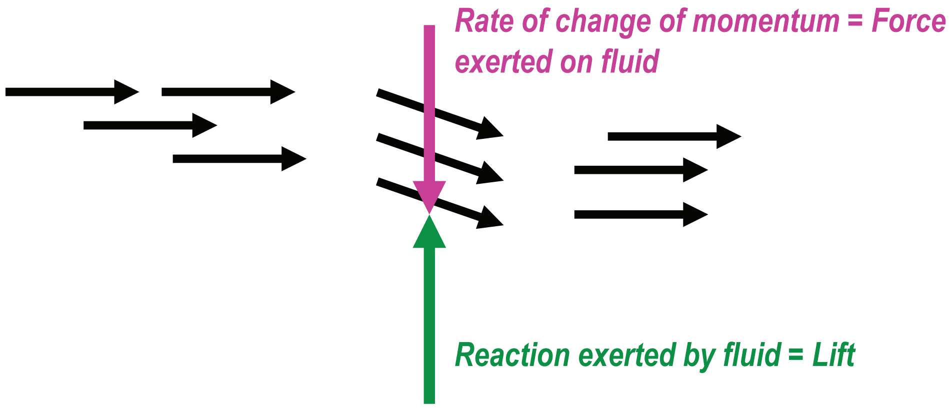


Figure 1.6: Lift Generation

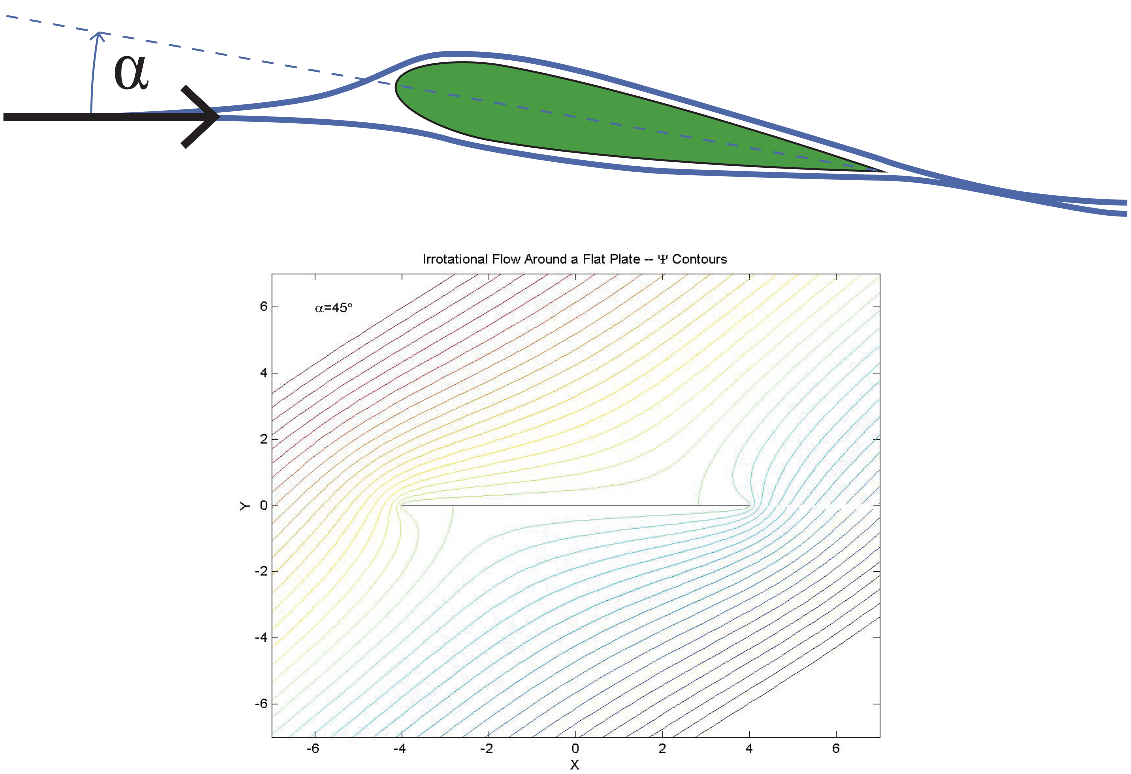
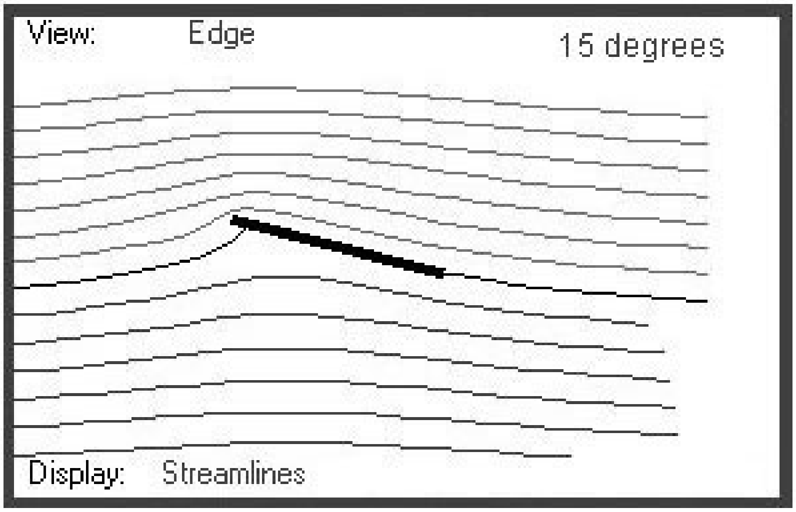


Figure 1.7: Airfoil At Angle of Attack: Streamline Pattern



http://www.windsofkansas.com/AZ2Fig5.jpg

Figure 1.8: Flat Plate At Angle of Attack: Streamline Pattern

How to turn the flow and generate lift: Camber

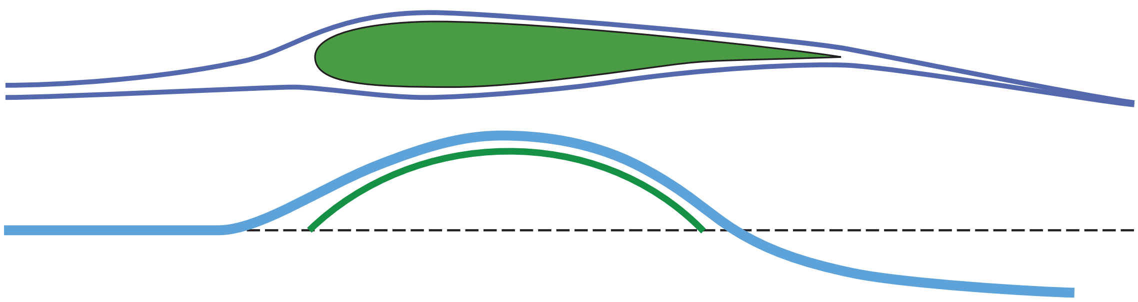


Figure 1.9: How To Generate Lift Using Camber

1.7 Evolution of the Airfoil Shape

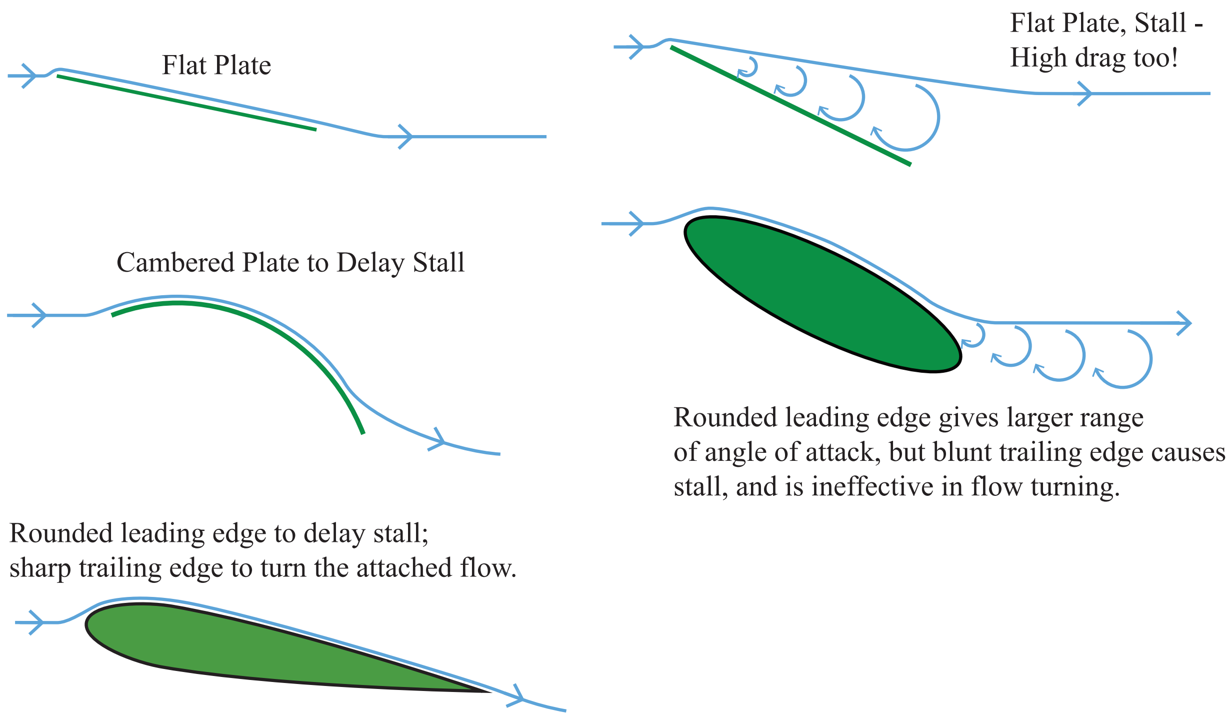


Figure 1.10: Evolution of the Airfoil Shape

1.8 Vortex-Induced Lift

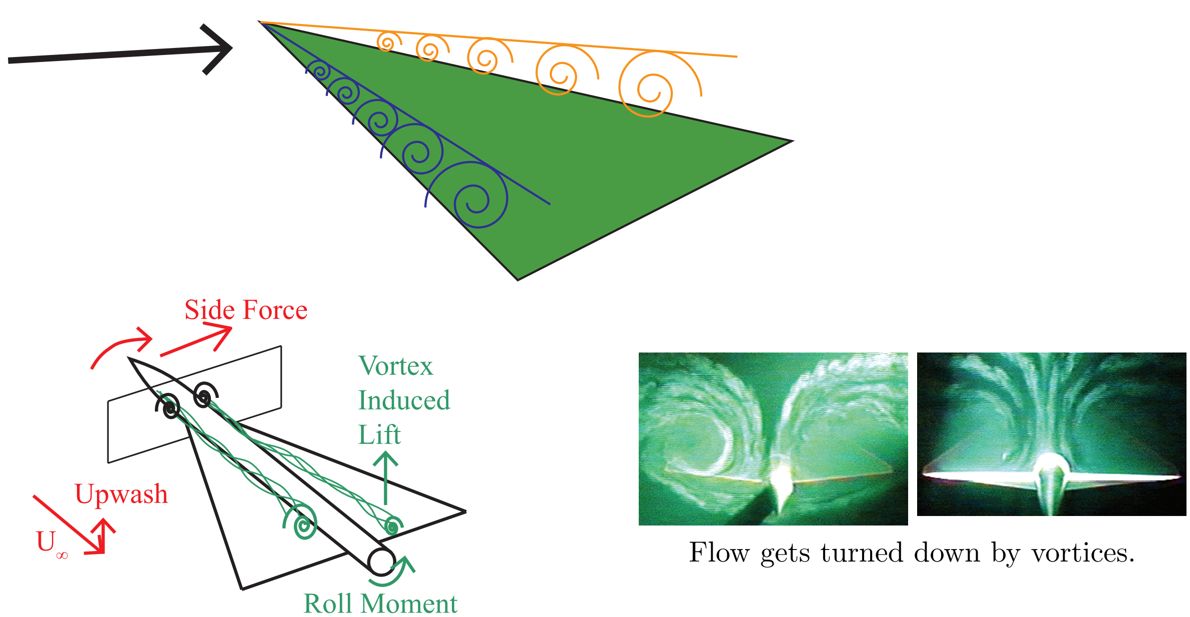


Figure 1.11: Vortex-Induced Lift

1.9 Vortex-Induced Lift and Delta Wings

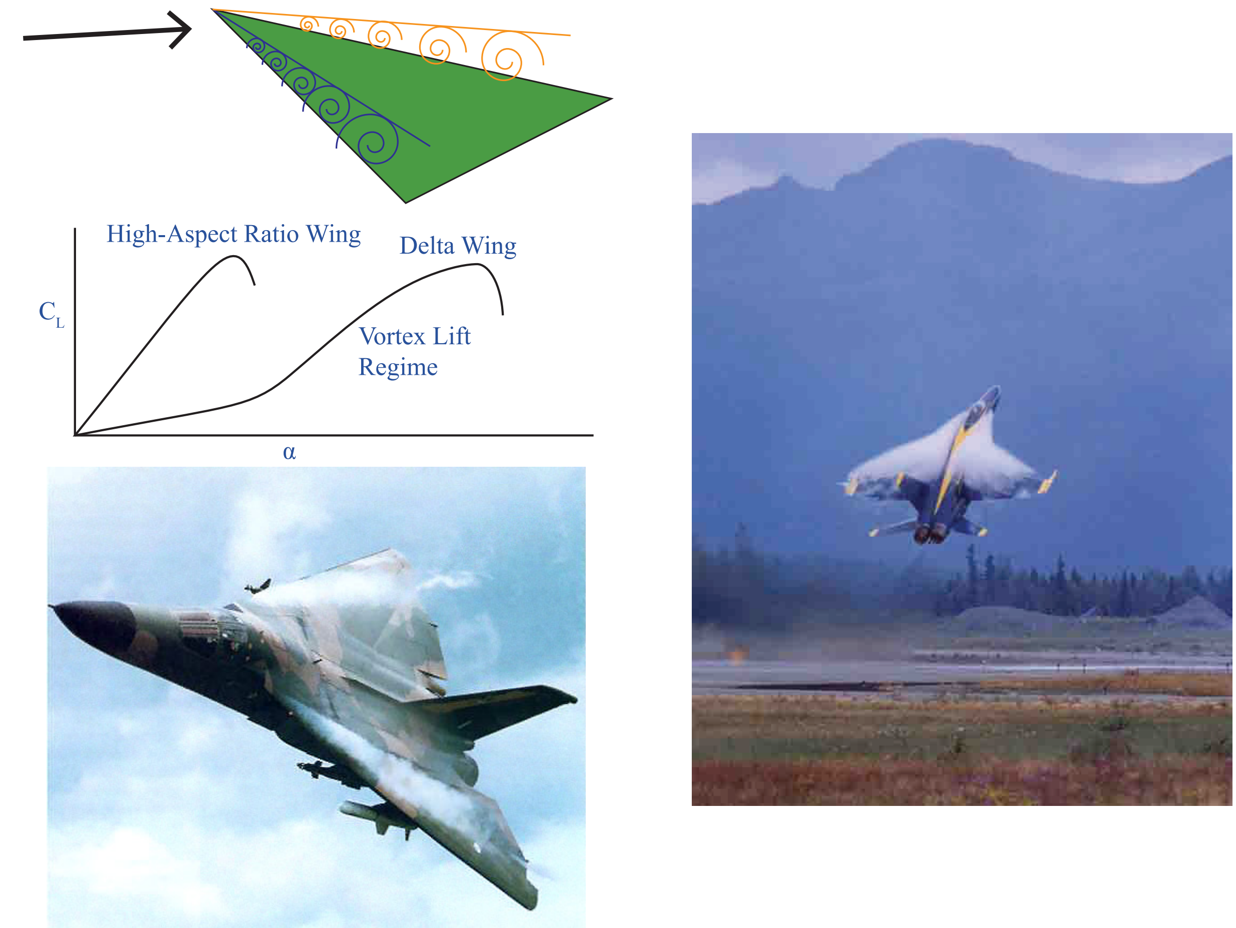


Figure 1.12: Vortex-Induced Lift and Delta Wings

1.10 Airfoil (British: "aerofoil")

"Airfoil" means "shape of a section of a wing." It is a two-dimensional concept.
Airfoils cannot fly: wings fly.
Airfoil properties are used to calculate and design wing properties.
Airfoil lift coefficient C_L varies with angle of attack α.
If the airfoil is cambered, the lift coefficient is positive even at zero angle of attack, and reaches zero only at some negative value of angle of attack: this is called the "zero-lift angle of attack," α_0.
As the camber is increased, α_0 becomes more negative.
Thus airfoil lift coefficient is

$$C_L = \frac{dC_L}{d\alpha}(\alpha - \alpha_0) \tag{1.7}$$

The lift-curve slope $dC_L/d\alpha$ tells us how much lift increase we can expect to get from increasing angle of attack, from a given angle of attack.
$dC_L/d\alpha \leq 2\pi$, where α is in radians.

1.11 Airfoils vs. Wings

$$L' = \text{Lift per unit span} = q_\infty c C_L \tag{1.8}$$
$$D' = \text{Drag per unit span} = q_\infty c C_D \tag{1.9}$$

c: Airfoil chord, which is the line joining leading edge and trailing edge.

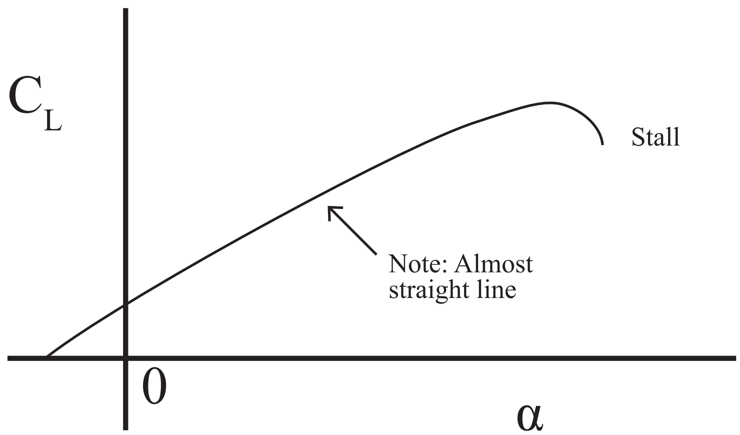


Figure 1.13: C_L curve

1.12 Lift Curve Slope of an Airfoil

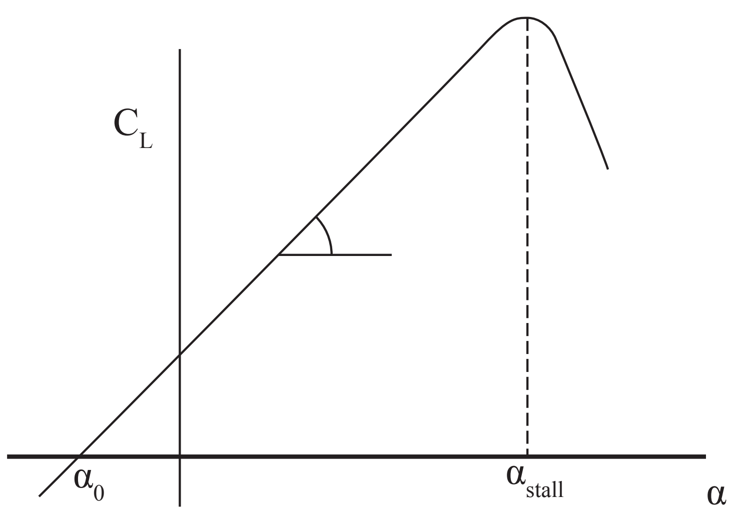


Figure 1.14: Lift Curve Slope of an Airfoil

"Thin-airfoil theory" in low-speed flows where Mach number is close to 0, proves that:

$$\frac{dC_L}{d\alpha} \leq 2\pi \text{ in low-speed flows} \tag{1.10}$$

1.13 Exercise: Lift coefficient

The angle of attack of an airfoil is 12 degrees. The lift curve slope is 5.8 per radian. Zero-lift angle of attack is -2 degrees. Find the lift coefficient.

If the air density is 1/10 of sea-level standard, and the temperature is 20 °C higher than the standard sea-level, flight speed is 100 m/s and wing planform area is 30 m^2, find the lift.

Sea level standard temperature is 15 °C or 288 K.

1.14 Pressure Coefficient

The pressure coefficient is a way to express the pressure with respect to some reference pressure, as a "dimensionless" quantity.

$$C_p = \frac{p - p_\infty}{0.5\rho U_\infty^2} = \frac{p - p_\infty}{q_\infty} = 1 - \left(\frac{U}{U_\infty}\right)^2 \tag{1.11}$$

$C_p = 0$ indicates the undisturbed freestream value of static pressure.
$C_p = 1$ indicates a stagnation point.
$C_p < 0$ indicates a suction region.

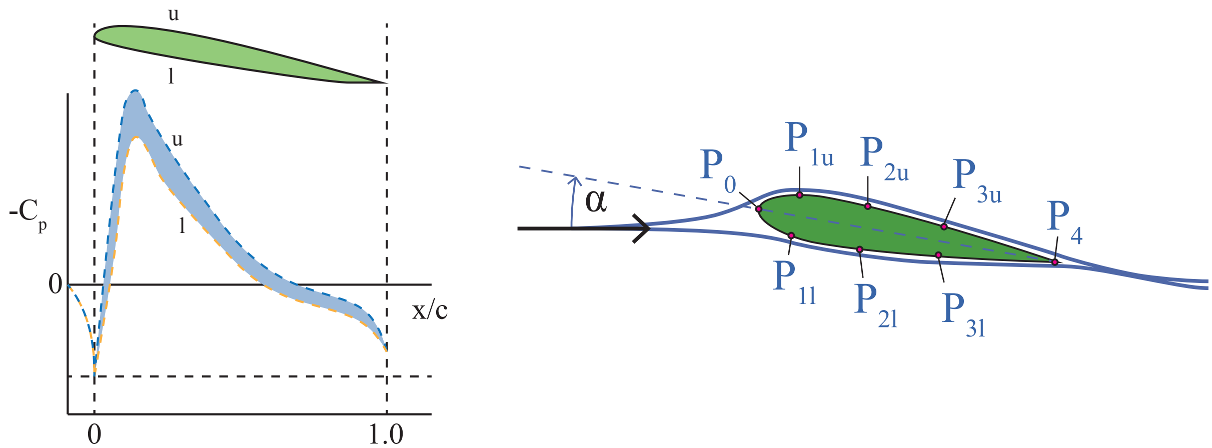
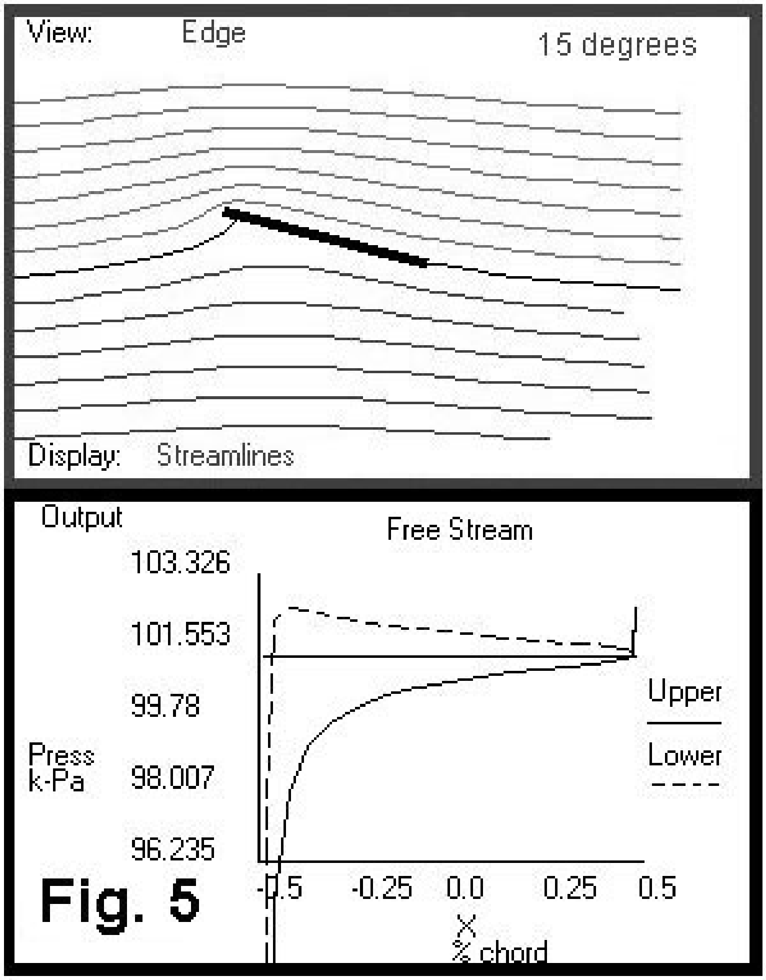


Figure 1.15: Chordwise pressure distribution over an airfoil in low-speed flow

1.15 Flat Plate Streamlines and Pressure Distribution



http://www.windsofkansas.com/AZ2Fig5.jpg

Figure 1.16: Flat Plate Streamlines and Pressure Distribution

1.16 Exercise: Pressure Coefficient

- What is the pressure coefficient at the stagnation point of an airfoil section?
- What is the pressure coefficient on a flat surface aligned with the freestream?
- C_p at the suction peak of an airfoil is -1.2. What is the pressure there as a percentage of the freestream static pressure?
- What is the velocity at this point, as a percentage of the freestream velocity?

1.17 Drag

Ideally, nothing happens to the fluid when an object goes through, except that it gets moved aside, or maybe turned a bit, but then recovers.

In practice, some effect always remains. Some flow gets pulled along the direction of the object until its kinetic energy is dissipated as heat. Or there is a region of swirling flow left behind, whose energy is also dissipated as heat. The force resulting from these things is drag. It acts along the freestream direction.

The drag is given by:

$$D = q_\infty S C_D \tag{1.12}$$

Lift to Drag Ratio:

$$L/D = \frac{C_L}{C_D} \tag{1.13}$$

We want our airplanes to have as high L/D as possible!

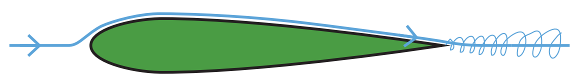


Figure 1.17: Swirling flow left behind the airfoil

The profile drag of an airfoil of chord 1 unit is about the same as that of a circular cylinder whose diameter is only 0.005 unit.

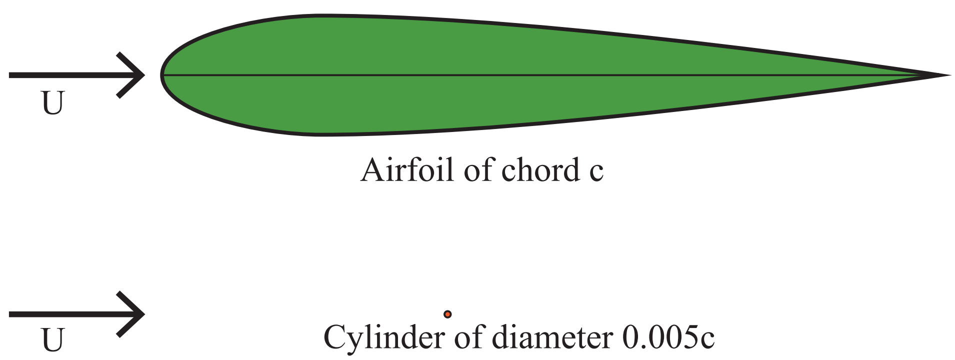


Figure 1.18: Profile Drag of a Cylinder vs. Airfoil

Streamlining: At high Reynolds numbers, the drag of an airfoil with chord c is only as much as that of a cylinder of diameter $0.005c$. From Covert, Eugene C., "The Legacy of the Wright Brothers and Its Future," 1997 Wright Brothers Lecture.

But we want our space reentry capsules and parachutes to have very low L/D.

http://ares.jsc.nasa.gov/HumanExplore/Exploration/EXLibrary/images/Pics/MarsScenario1/09Chute.gif

http://serc.carleton.edu/images/NAGTWorkshops/careerprep/jobsearch/Parachute_300.jpg

Figure 1.19: Examples of low L/D

1.18 Drag Coefficient

The drag is given by $D = 0.5\rho U_\infty^2 S C_D$.
The drag coefficient in low-speed flow is composed of 3 parts:

$$C_D = C_{D0} + C_{D\text{friction}} + C_{Di} \tag{1.14}$$

when C_{D0} is the parasite drag, which is independent of lift. It is usually due to the losses of stagnation pressure which occur when part of the flow separates somewhere along the wing or body surface. In high speed flight, the effect of shocks and wave drag must be added to this, and becomes dominant source of drag.

Example:
C_{D0} of a small airliner is 0.018. Wing aspect ratio is 6. Assume spanwise efficiency is 1.0. Lift coefficient is 0.5. Find the total drag coefficient.
If the density is 1 kg/m^3 and speed is 200 m/s, find the drag.

1.19 Drag Calculation

$$\text{Drag} = (\text{Dynamic Pressure}) \times (\text{Reference Area}) \times (\text{Drag Coefficient}) \tag{1.15}$$

$$D = q_\infty S C_D \tag{1.16}$$

$$\frac{D}{q_\infty} = SC_D \text{ is called "Equivalent Flat Plate Area"} \tag{1.17}$$

Note: Unlike lift, which people associate with the wings and hence planform area S, everything produces drag. So when the drag of each component is calculated, the drag coefficient of that is based on the "referenced area" for that component. For instance, for a fuselage the reference area is the cross-section area.

Thus when everything is summed up, you use the concept of "Equivalent Flat Plate Area" rather than C_D to make sure we don't use different reference areas.

1.20 Example: Equivalent Flat Plate Area

Subscript "0" indicates "zero-lift" value.

Component	C_{D0}	Area	D_0/q, ft^2
Fuselage + Engine Nacelles	0.03	C/S area: 64 ft^2	?
Wings	0.003	Planform area: 120 ft^2	?
Tails	0.004	Tail planform area: 15 ft^2	?
Landing Gear	1.2	Strut diameter$\times$Length = 2/12$\times$5	?

Total D_0/q [ft^2] of the airplane = ?
C_{D0} based on planform area = ?

Let's say that the airplane is flying at minimum drag: Lift-induced drag = Profile drag. Let's say that the aircraft weight is 5000 lbs. What is the L/D?

1.21 Lift-Induced Drag and Aspect Ratio

At the ends of the wings, the pressure difference between the upper and lower sides is lost, as the flow rolls up into a vortex.

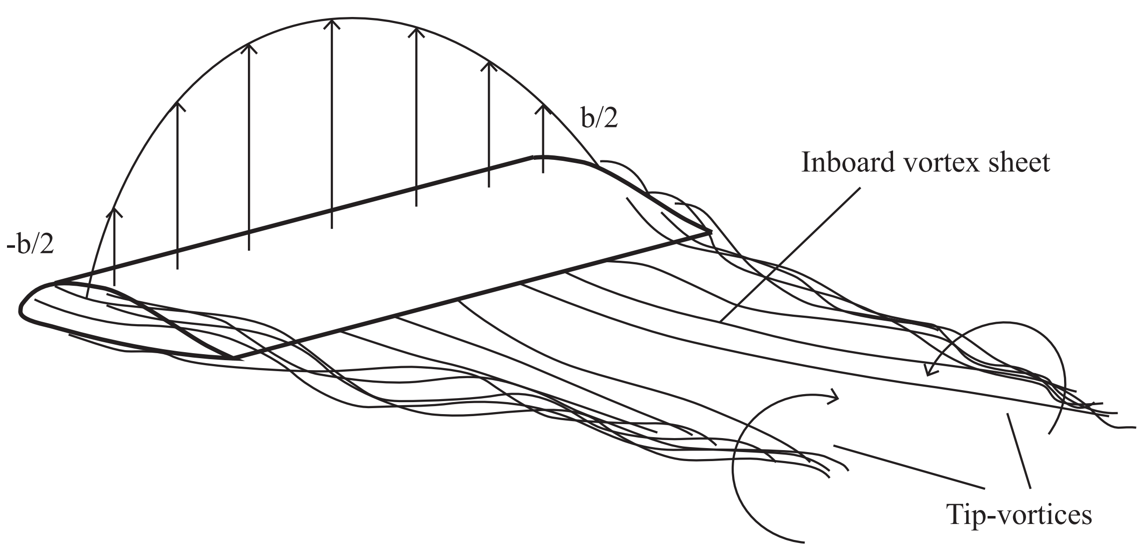


Figure 1.20: Tip vortices

The aspect ratio of a wing is defined as:

$$AR = \frac{b^2}{S} \tag{1.18}$$

where b is the wing span and S is the wing planform area.

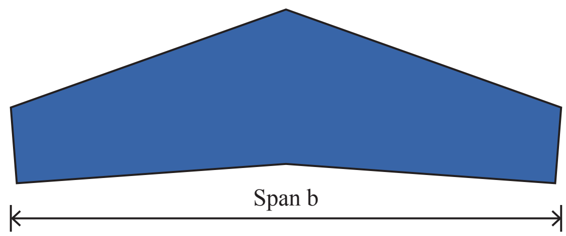


Figure 1.21: Wing span b

1.22 Effects of Finite Aspect Ratio

1. The overall lift is reduced, relative to the airfoil lift value predicted for a section of an infinite wing.
2. The lift vector is tilted back, so that an "induced drag" is created.

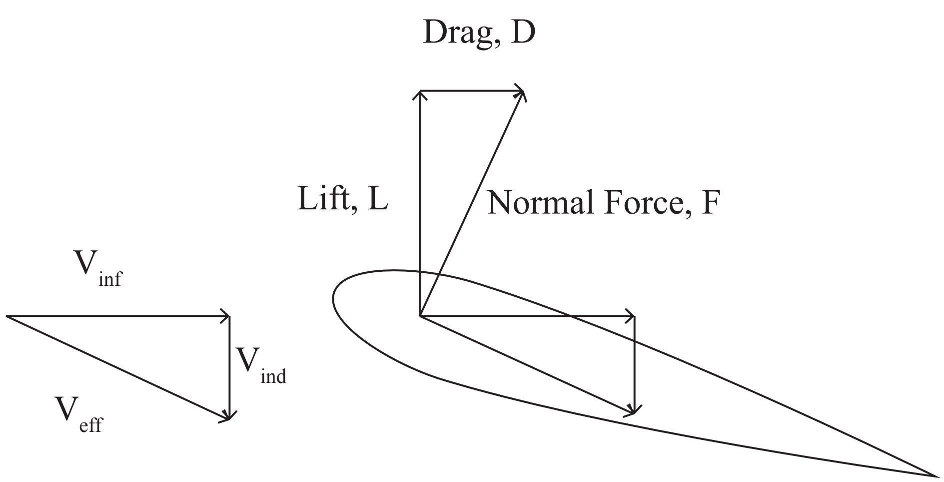


Figure 1.22: Induced drag

Both of these (usually undesirable) effects are reduced by increasing the aspect ratio of the wing.

1.23 Points to Note on Finite Wings

1. Since induced drag is directly related to the lift it can be calculated by the same mathematical formulation used to calculate lift.
2. Does not require consideration of viscosity.

3. No induced drag on 2-D airfoils under steady conditions.

4. At finite Aspect ratios,

$$C_{Di} = \frac{C_L^2}{\pi(AR)e} \tag{1.19}$$

5. Ideal elliptic lift distribution implies minimum induced drag, i.e. spanwise efficiency $e = 1$.

Note: In the 2-D limit (airfoil) there is no lift-induced drag in incompressible flow. There is always "profile" drag due to viscous effects (friction drag) and flow separation (pressure drag).

1.24 Effect of Aspect Ratio on Wing Lift Curve Slope

$$\frac{dC_L}{d\alpha} \equiv a = \frac{a_0}{1 + \left(\frac{a_0}{\pi(AR)}\right)(1+\tau)} \tag{1.20}$$

$$(1+\tau) \approx \frac{1}{e} \tag{1.21}$$

$$a_0 \equiv \frac{dc_l}{d\alpha} \tag{1.22}$$

1.25 Typical L/D values of different types of aircraft

- Transport aircraft: (L/D) 16 - 20
- Fighters (L/D) cruise 10 - 16
- Supersonic transport: (L/D) 11
- Hypersonic aircraft: (L/D) 1 (lift is a minor problem here)

1.26 Example on L/D and Wing Loading

Example:
An aircraft has a wing loading (W/S) of 130 pounds per square foot (6233 N/m^2), aspect ratio of 7.667, and wing span of 60.96 m. We'll assume that its spanwise efficiency factor will be 0.99. Let's assume that the profile drag coefficient is given by $C_{D0} = 0.015$. Thus, for maximum Lift-to-Drag ratio (minimum drag, and the lift is always equal to the weight for straight and level flight), $C_{Di} = C_{D0} = 0.015$.

The corresponding C_L is calculated as 0.598, and the dynamic pressure is 10423 N/m^2. At 11,000 meters in the Standard Atmosphere, density is 0.36 kg/m^3, so that the flight speed is 240.64 m/s.

Note: In practice, the C_{D0} might change with flight Mach number, for high-speed flight. This is not taken into account in the above.

1.27 Aerodynamics Summary

Lift is force perpendicular to the flow direction, due to pressure differences across surfaces. 3 ways of generating lift are:

a) angle of attack

b) camber

c) vortex-induced lift

An infinite (2-dimensional) wing is entirely described by its airfoil section. Finite wings have less lift than corresponding span-lengths of an infinite wing at the same angle of attack, and also have lift-induced drag. The total drag is composed of profile drag, which does not vary with lift, and induced drag, which rises as the square of the lift coefficient.
To fly an airplane of a given weight, straight and level, the condition for minimum drag (maximum lift-to-drag ratio) is that the profile drag coefficient is the same as the induced drag coefficient.

1.28 Speed for Minimum Drag

Total drag is composed of a part which depends on lift, and one that does not.

$$D = D_0 + D_i = (C_{D0} + C_{Di})\left(0.5\rho U_\infty^2 S\right) \tag{1.23}$$

$$D = \left(C_{D0} + \frac{C_L^2}{\pi(AR)e}\right)\left(0.5\rho U_\infty^2 S\right) \tag{1.24}$$

Let us consider what it takes to keep L = W.

$$W = L = q_\infty S C_L \tag{1.25}$$

$$C_L = \frac{W}{q_\infty S} \tag{1.26}$$

So

$$D = q_\infty S C_{D0} + \left(\frac{W}{S}\right)^2 \frac{1}{\pi(AR)e}\frac{S}{q_\infty} \tag{1.27}$$

$$\frac{dD}{dq_\infty} = SC_{D0} - \left(\frac{W}{S}\right)^2 \frac{1}{\pi(AR)e}\frac{S}{q_\infty^2} = 0 \tag{1.28}$$

i.e. $C_{D0} = C_{Di}$ or, Minimum Total Drag = Twice Zero-lift Drag

... This is a remarkable result. It means that:
AIRCRAFT, UNLIKE OTHER FORMS OF TRANSPORTATION, HAVE A DEFINITE SPEED FOR MINIMUM DRAG!

To fly an airplane of a given weight, straight and level, the condition for minimum drag (maximum lift-to-drag ratio) is that the profile drag coefficient is the same as the induced drag coefficient.

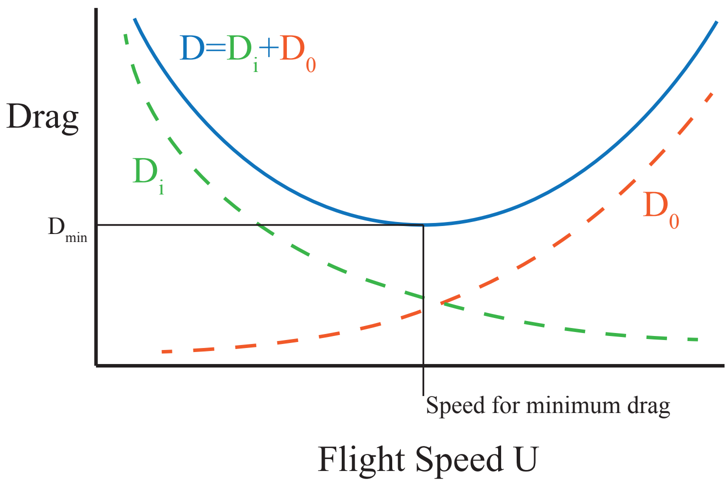


Figure 1.23: Profile Drag, Induced Drag and Total Drag versus Speed

1.29 Kutta-Jowkowsky Theorem

Derived from Newton's 2nd law.
Derived by observing that there are 2 ways that lift can be explained:

1. Due to circulation (flow turning)
   Lift per unit span $\vec{L}' = \rho \vec{U}_\infty \times \vec{\Gamma}$. Here $\vec{\Gamma}$ is the Bound Circulation.

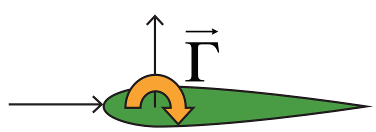


Figure 1.24: Kutta-Jowkowsky Theorem

2. Due to compression / expansion of the flow (density changes due to velocity changes):
   This cannot happen in incompressible flow, which is what we study in this course.

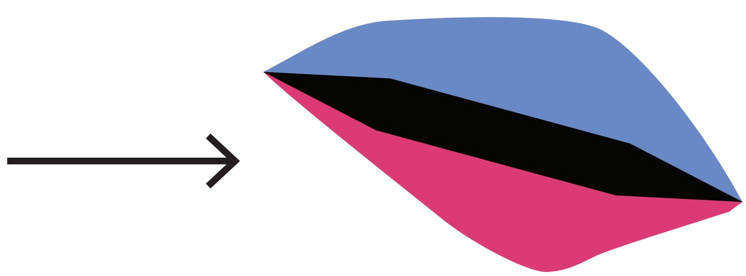


Figure 1.25: Compression and expansion of the flow

Chapter 2

Introduction to Fluid Mechanics

2.1 Types of Fluid Motion

Fluids, like most other forms of matter, are made up of tiny particles (molecules), which are separated by large spaces.

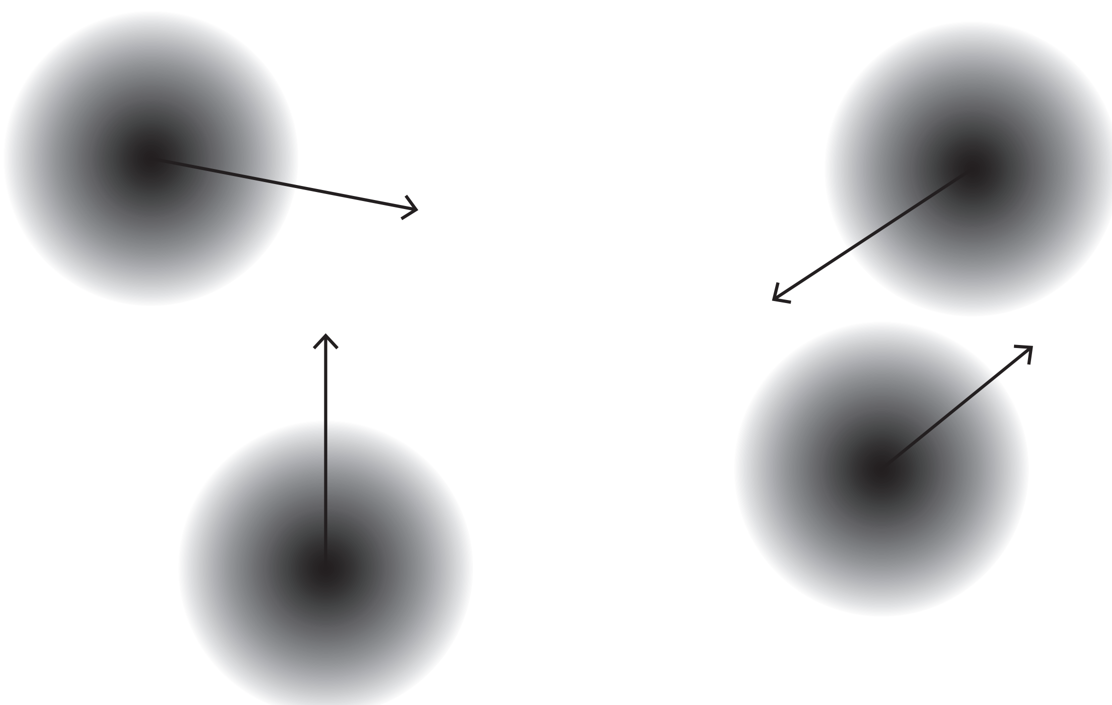


Figure 2.1: Molecular Kinetics

How many molecules are there in a cubic centimeter?

Avogadro's # is 6.022115×10^{23}.
Gives # of atoms (or molecules) per gram-mole of the gas.

1 gram-mole of air is roughly 28.97 grams. Molecular weight $\hat{M}$ is 28.97 grams/mole, or kg/kg-mole.

Density of air (mass/volume) can be found from the Perfect Gas Law. Let's say, we have air at sea level standard conditions. Pressure is 1.01325×10^5 N/m^2, and temperature is 288 K.

$$\rho = \frac{P}{RT} \tag{2.1}$$
$$= \frac{P\hat{M}}{\hat{R}T} \tag{2.2}$$
$$= \frac{101325 \times 28.97}{8313 \times 288} \tag{2.3}$$

Density is 1.225 kg/m^3.
So how many grams in a cubic centimeter? 1.225×10^{-3}.
How many molecules? $\frac{1.225 \times 10^{-3}}{28.97} \times 6.0221415 \times 10^{23} = 2.55 \times 10^{19}$.

So we can treat fluid as a "continuum":
a medium which is pretty uniform in properties at the smallest scales of interest to us.

Fluid Motion

In the rest of this course we will treat fluids as "continuous media," and forget about those molecules zipping about at random.

The smallest "packet" of fluid which we will now consider will have many millions of molecules. The motion of these packets is the net motion of all those molecules zipping around within the packet (think of a school bus moving at 5 mph with thirty middle-schoolers conducting a football game inside using each other's backpacks.)

Streamline

A streamline is a curve whose tangent at any point is in the direction of the velocity vector at that point.
NOTE: if the flow is not steady, the streamline has little use or significance.

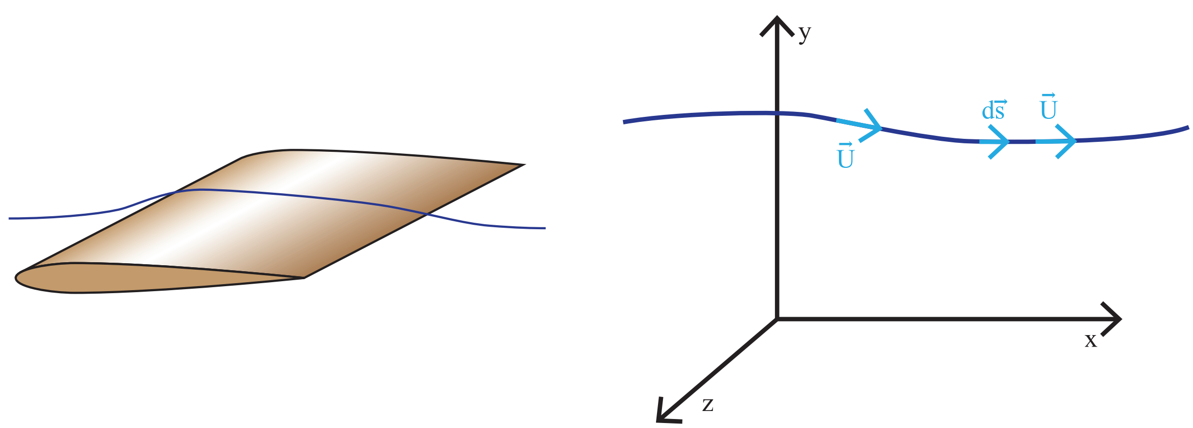


Figure 2.2: Streamline

If $d\vec{s}$ is an elemental vector along the streamline,

$$d\vec{s} = dx\vec{i} + dy\vec{j} + dz\vec{k} \tag{2.4}$$
$$\vec{U} = u\vec{i} + v\vec{j} + w\vec{k} \tag{2.5}$$

$$d\vec{s} \times \vec{U} = \begin{vmatrix} i & j & k \\ dx & dy & dz \\ u & v & w \end{vmatrix} \tag{2.6}$$

$$\Rightarrow i(wdy - vdz) + j(udz - wdx) + k(vdx - udy) = 0 \tag{2.7}$$

Equation to a streamlines is:

$$wdy = vdz \qquad \frac{dz}{dy} = \frac{w}{v} \tag{2.8}$$

$$udz = wdx \qquad \frac{dz}{dx} = \frac{w}{u} \tag{2.9}$$

Four Basic Types of Fluid Motion

As a "packet" of fluid moves along, a combination of 4 things can happen to it: **translation**, **dilatation**, **rotation** and **shear**. Thus any fluid motion can be described as some combination of these.

1) Translation: motion of the center of mass
   This is characterized by the velocity

$$\vec{U} = u\vec{i} + v\vec{j} + w\vec{k} \tag{2.10}$$

$$\rho = \frac{P}{RT} \tag{2.11}$$

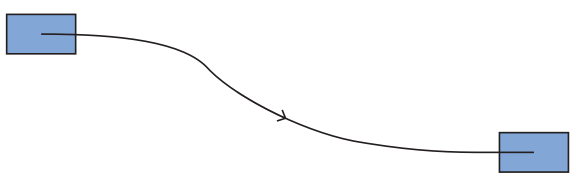


Figure 2.3: Translation

2) Dilatation: volume change

$$\nabla \cdot \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \tag{2.12}$$

   Also called "Divergence of the velocity vector."

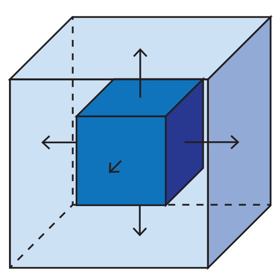


Figure 2.4: Dilatation

3) Rotation: about one, two or three axes

$$\vec{\omega} = \frac{1}{2}\left[\left(\frac{dw}{dy} - \frac{dv}{dz}\right)\vec{i} + \left(\frac{du}{dz} - \frac{dw}{dx}\right)\vec{j} + \left(\frac{dv}{dx} - \frac{du}{dy}\right)\vec{k}\right] \tag{2.13}$$

$$\text{vorticity}\, \zeta = 2\omega = \nabla \times \vec{U} \tag{2.14}$$

Note that rotation is a vector. We usually use a quantity called "vorticity" ζ which is twice the rotation vector, to describe the amount of rotation in flows.

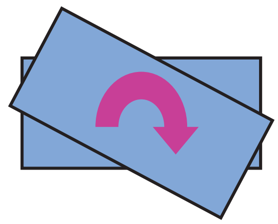


Figure 2.5: Rotation

4) Shear Strain
Strain is defined as a change in length per unit length. Rate of strain is thus given by change in velocity per unit length. Shear exists when there is a gradient of velocity along the direction perpendicular to streamline.

$$\epsilon_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \qquad \epsilon_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \qquad \epsilon_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \tag{2.15}$$

The quantity "strain rate" has nine components. They are:
ϵ_{xx}, ϵ_{yy}, ϵ_{zz} These are "normal strain rate" components.
ϵ_{xy}, ϵ_{yz}, ϵ_{zx} These are "shear strain rate" components.
ϵ_{yx}, ϵ_{zy}, ϵ_{xz} These are equal to the corresponding "shear strain rate" components above.

2.2 Circulation $\Gamma = -\oint_c \vec{U} \cdot d\vec{l}$

Defined as an integral around a closed contour "c." The negative sign is included such that positive circulation on a body corresponds to positive lift, and the integral is evaluated counter-clockwise. From the preceding discussion, we see that will be zero unless there is some vorticity, contained within the contour.

Γ is an extremely useful quantity: it helps us calculate lift, vortex strength, etc.

Important points:
The circulation around a closed contour with net rotation and/or shear will be non-zero. It is, however, always possible to have a combination of rotation and/or shear that gives a zero circulation.

Note: In the above, the definition is based on a line integral around a contour, which is in a given plane. This works when we know the plane of the rotation, and that the flow is entirely 2-dimensional. To deal with general problems, we can consider Γ to be a vector aligned along the axis of rotation.

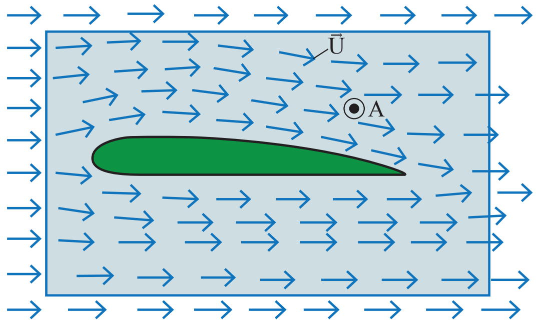


Figure 2.6: Circulation

2.3 Kutta-Jowkowsky Theorem (revisited)

Derived from Newton's 2nd law.
Derived by observing that there are 2 ways that lift can be explained:

1. Due to circulation (flow turning)
   Lift per unit span $\vec{L'} = \rho \vec{U}_\infty \times \vec{\Gamma}$. Here $\vec{\Gamma}$ is the Bound Circulation.

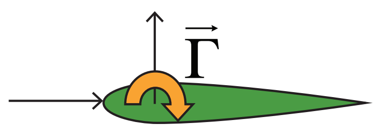


Figure 2.7: Kutta-Jowkowsky Theorem

2. Due to compression / expansion of the flow (density changes due to velocity changes):
   This cannot happen in incompressible flow, which is what we study in this course.

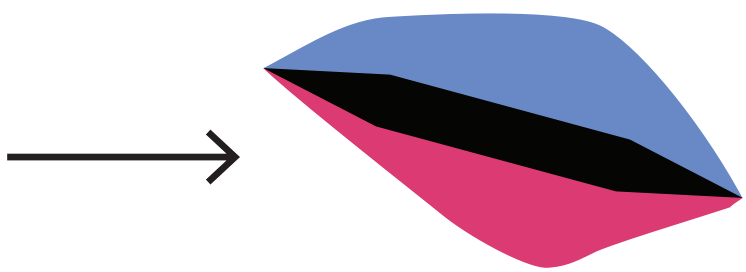


Figure 2.8: Compression and expansion of the flow

Chapter 3

Conservation Equations of Fluid Mechanics

3.1 Conservation of Mass

If we have a fluid going in and coming out of a given region of interest, (a "control volume"), we can say for sure that (what goes in per unit time) = (what comes out per unit time) + (what accumulates inside per unit time).

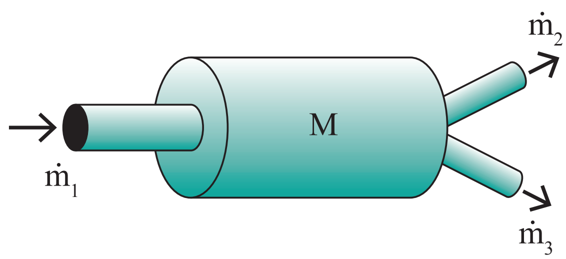


Figure 3.1: Conservation of Mass

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 + \left(\frac{\Delta M}{\Delta t}\right)_{\text{inside}} \tag{3.1}$$

$$\begin{aligned}\text{(Mass going in per second)} &= \text{(Sum of masses going out per second)} \\ &\quad + \text{(Mass accumulated inside in 1 second)}\end{aligned} \tag{3.2}$$

In general, mass may be going in and/or out everywhere across an (imaginary?) surface enclosing the space in which you are interested. Also, the velocity of the inflow/outflow may be nonuniform, and in some arbitrary direction.

$$\text{Mass flowing out of the surface of the control volume per unit time} = \oiint_S \rho(\vec{u}\cdot\vec{n})d\vec{S} \tag{3.3}$$

where $d\vec{S}$ is a small element of the total surface area S. An "integral" is a neat way of saying, "collect all the little bits and add them up."

Law of conservation of mass for a control volume becomes

$$\oiint_S \rho(\vec{u}\cdot\vec{n})d\vec{S} + \frac{\partial}{\partial t}\iiint_V \rho dV = 0 \tag{3.4}$$

"Conservation" of anything can be expressed this way, as we will see. Here the r.h.s. is zero: mass can't get converted to anything.

3.2 Example: Closed circuit wind tunnel

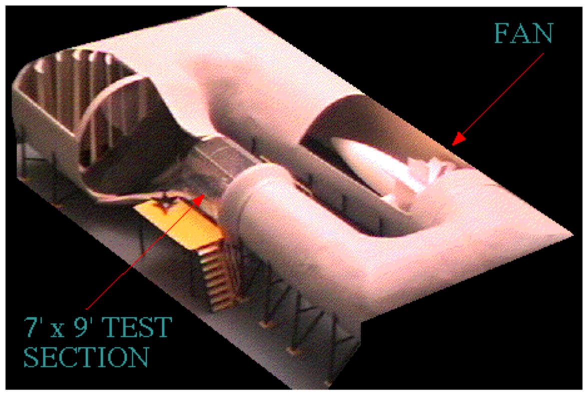


Figure 3.2: Closed-Circuit Low Speed Wind Tunnel

Settling Chamber velocity = 15 fps, uniform; steady. Chamber diameter: 20 feet. Test section is a 9-foot diameter duct. Our "control volume" has one face in the settling chamber, other face in the test section. Assume that no flow can escape out the sides of the control volume. "Steady" means that at any given point in the flow, the properties don't change as time changes.

$$\frac{\partial}{\partial t}(\text{anything}) = 0 \tag{3.5}$$

The "integral form of the mass conservation equation" becomes:

$$\oiint_S \rho(\vec{u}\cdot\vec{n})d\vec{S} = 0 \tag{3.6}$$

Subscript "1" refers to settling chamber face of control volume, subscript "2" refers to test section. Air cannot escape out of the sides of the "control volume," so the integral reduces to just

$$-\rho_1 A_1 U_1 = \rho_2 A_2 U_2 = 0 \tag{3.7}$$

Negative sign: Look at direction of vector normal to each face, with respect to the direction of U. Thus

$$U_2 = \frac{\rho_1 A_1 U_1}{\rho_2 A_2} \tag{3.8}$$

Since the density change can be neglected, and each area is a circle,

$$U_2 = \left(\frac{20}{9}\right)^2 \times 15 = 74.07\,\text{ft/s} \tag{3.9}$$

3.3 Conservation of Momentum

Newton's 2nd law of Motion: "Rate of Change of Momentum = Force"
or "Net force acting on a system = time rate of change of momentum of system"
Note units:

$$\begin{aligned}\text{Momentum} &= \text{Mass} \times \text{Velocity} & (3.10)\\ &= \text{Density} \times \text{Volume} \times \text{Velocity} & (3.11)\\ &= \text{Density} \times \text{Area} \times \text{Distance} \times \text{Velocity} & (3.12)\end{aligned}$$

$$\text{Momentumperunittime} = \text{Density} \times \text{Velocity} \times \text{Velocity} \times \text{Area} \tag{3.13}$$

What is being conserved? "momentum per unit volume". So the conservation equation has 2 terms on the left hand side:

1. "rate of change of the quantity, integrated over the whole control volume". This is the "unsteady term".

2. "net outflow per unit time of the quantity across the surfaces of the control volume, integrated over the whole control surface". This is the convection term.

On the right hand side, we have surface and volume integrals of the quantities to which our "conserved quantity" may have changed. In this case, when momentum disappears, the rate of change of momentum produces forces of various kinds.

$$\iint\limits_{\text{C.S.}} (\rho\vec{u})(\vec{u}\cdot\vec{n})dA + \frac{\partial}{\partial t}\iiint\limits_{\text{C.V.}} (\rho\vec{u})dV = \iiint\limits_{\text{vol}} \text{force} \tag{3.14}$$

3.4 Body and Surface Forces

a) Body forces

$$\text{Gravitational force per unit volume} = \rho\vec{g} \tag{3.15}$$

(Important for liquid flows, and buoyant gas flows).
Electromagnetic force: it would be something like $\rho(\vec{B} \times \vec{\epsilon})$.
Important in ion propulsion, spark plugs, plasmas. Generally, we neglect body forces in aerodynamics.

b) Surface forces
Pressure is the result of molecules, flying about at random, crossing (or colliding with) the surface, thus transferring momentum. So it acts normal to the surface. It must be related to # of molecules/volume (thus related to density), and speed of random motion of molecules (thus related to temperature). Thus, pressure is related to the product of density and temperature.

This relation is a simple proportionality (multiply by a constant) where R is the gas constant. This is called the "perfect gas law."

So force due to pressure is $-\oiint_S (p\vec{n})dS$ because pressure acts on the surface, opposite to $\vec{n}$.

3.5 Shear Stress vs. Rate-of-Strain Relations

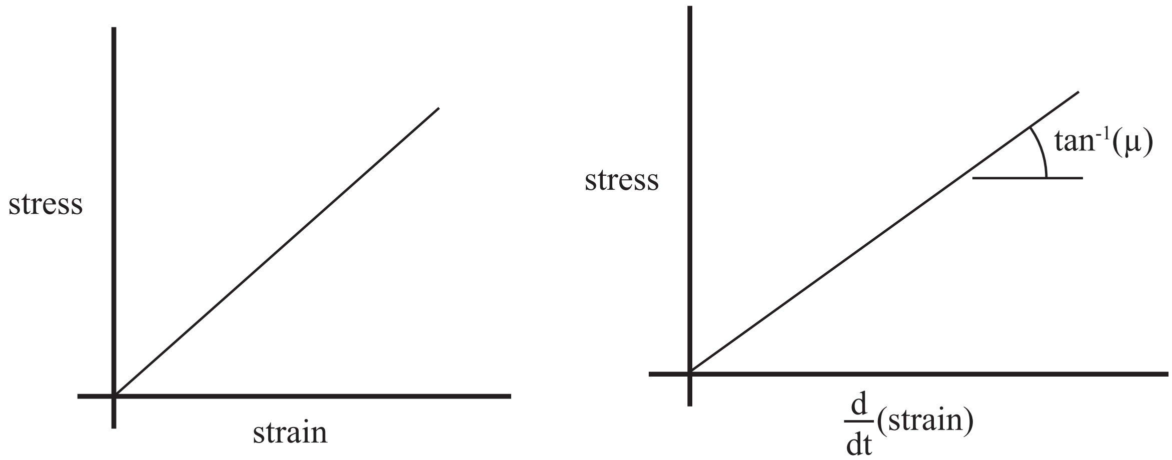


Figure 3.3: Stress and Strain

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad (3.16)$$

$$\sigma_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \quad (3.17)$$

$$\sigma_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \quad (3.18)$$

The direction is indicated by the second subscript. The units of stress are Force/Area, such as N/m^2, psi, or psf.

These can be written in compact form. Here u is used to represent any of the velocity components, and x is used to represent any of the spatial coordinates. i and j representing x,y,z in turn.

$$\sigma_{ij} = \mu\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) \quad (3.19)$$

The normal stress, on the other hand, is simply the pressure.

3.6 Normal Stress

Here "normal" is a fashionable term for "perpendicular to a surface," as opposed to "tangential," not as opposed to "abnormal"...
The normal stress is simply due to the pressure.

$$\sigma_{xx} = -p\vec{i} \quad (3.20)$$

$$\sigma_{yy} = -p\vec{j} \quad (3.21)$$

$$\sigma_{zz} = -p\vec{k} \quad (3.22)$$

The unit vectors are included to show the direction of each of these stress components.

Don't fluids resist the strain rate due to compression and expansion?
They do, and a relation between normal stress and rate of normal strain can be written, just like

the shear strain relations on the previous slide, and added on to the above expressions. However, in gases, the coefficient of resistance to normal rate of strain is very small, so that this "**bulk viscosity**" does not become significant unless the strain rate is extremely high.

This occurs, for instance, across a shock wave, where velocity decreases by a large factor in the space needed for a few (under 10) collisions between molecules.

In most other situations, it is a usual assumption that "**Bulk Viscosity is Zero**" in a gas so that the normal stress is attributed to just the pressure.

3.7 Energy Equation

In addition to mass and momentum, we can use the fact that "**energy is neither created nor destroyed, but can change form.**"

The "conserved quantity" is energy per unit volume. The lhs has the terms describing time rate of change, and "flux" across the surfaces of the control volume. The rhs has the terms describing what changes the energy in the control volume: work and heat transfer. We use the first law of thermodynamics, which says that if you do work or release heat, your energy is exhausted. We will leave detailed study of this equation to the course on compressible flow and thermodynamics. The measure of total energy in a flow is the "stagnation enthalpy," which is related to the stagnation value of temperature. So the energy equation reduces to: "**Stagnation temperature goes up if work or heat are added to the flowing fluid**."

In steady flow, with no work being added, and no heat being added (adiabatic flow), the rhs is zero, so that the energy equation reduces to "**Stagnation Temperature is Constant**":

$$T_0 = T + \frac{u^2}{2c_p} = \text{constant} \tag{3.23}$$

The c_p here is not pressure coefficient, but "specific heat" at constant pressure.

$$c_p = \frac{R\gamma}{\gamma - 1} \tag{3.24}$$

For diatomic gases such as air, at usual temperatures that we encounter in low speed aerodynamics, the "gamma" has the value 1.4. Thus $c_p = 3.5R$.

3.8 Converting the Integral Form to the Differential Form of the Conservation Equations

In this section we will first convert the integral conservation equations to a form suitable to apply at each point, so that we can track changes from one point to another.

So we have to go from the "integral form" over a control volume, to a "differential form" which deals with small changes from point to point.

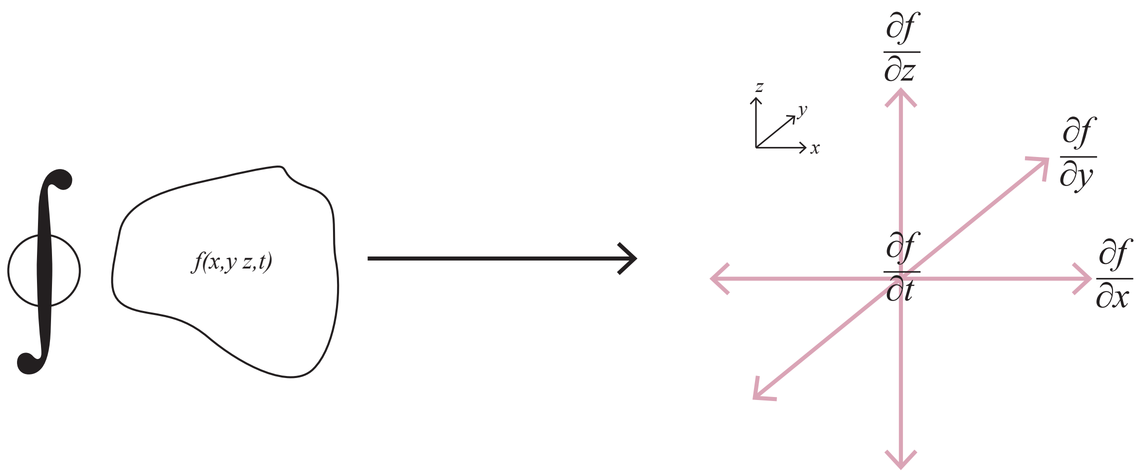


Figure 3.4: Integral over a region vs. gradient around a point

3.9 Relations between integrals over lines, surfaces and volumes

Why? Because we would like to bring everything under the same integral sign, set the whole integral to zero, and relate the quantities inside, thus getting rid of all the integrals:

$$\oiint_S (\ )dS \qquad \iiint_V (\ )dV \qquad \oint_C (\ )dl \tag{3.25}$$

Stokes' Theorem

Relates line integral over a closed contour to surface integral over the surface enclosed by the contour

$$\oint_C \vec{u}\cdot d\vec{l} = \oiint_S (\nabla\times\vec{u})\cdot d\vec{S} \tag{3.26}$$

This can also be written with the area element dS as a scalar, and the unit normal vector indicating its orientation.

$$\oint_C \vec{u}\cdot d\vec{l} = \oiint_S (\nabla\times\vec{u})\cdot \vec{n}dS \tag{3.27}$$

Divergence Theorem

Relates surface integral over a control surface to the volume integral over the volume enclosed by the control surface.

$$\oiint_S (\vec{u}\cdot\vec{n})dS \equiv \iiint_V (\nabla\cdot\vec{u})dV \tag{3.28}$$

Gradient Theorem

Relates surface integral of a scalar a control surface to the volume integral of the gradient of the scalar over the volume enclosed by the control surface.

$$\oiint_S p d\vec{S} \equiv \iiint_V (\nabla p)dV \tag{3.29}$$

3.10 Identity on the Gradient of a Product of a Vector and a Scalar

$$\nabla \cdot (\rho\vec{u}) \equiv \rho\nabla \cdot \vec{u} + \vec{u} \cdot \nabla\rho \tag{3.30}$$

3.11 Substantial Derivative

$$\frac{D}{Dt}(\,) \equiv \frac{\partial}{\partial t}(\,) + (\vec{u} \cdot \nabla)(\,) \tag{3.31}$$

The first term on the rhs is the "local" or "unsteady" term. The second is the "convective" term.

$$\frac{D}{Dt}(\,) \equiv \underbrace{\boxed{\frac{\partial}{\partial t}(\,)}}_{\text{“local” or “unsteady”}} + \underbrace{\boxed{u\frac{\partial}{\partial x}(\,) + v\frac{\partial}{\partial y}(\,) + w\frac{\partial}{\partial z}(\,)}}_{\text{“convective”}} \tag{3.32}$$

The rate of change $D()/Dt$ is for two reasons:

1. Things are changing at the point through which the element is moving (unsteady, local)
2. The element is moving into regions with different properties.

3.12 Differential Form of the Continuity Equation

$$\oiint_S \rho(\vec{u} \cdot \vec{n})dS + \frac{\partial}{\partial t}\iiint_V \rho dV = 0 \tag{3.33}$$

Use the Divergence Theorm $\oiint_S(\vec{u} \cdot \vec{n})dS \equiv \iiint_V(\nabla \cdot \vec{u})dV$:

$$\iiint_V \left(\frac{\partial\rho}{\partial t} + \nabla \cdot \rho\vec{u}\right) dV = 0 \tag{3.34}$$

We can take the limiting case as V tends to 0 – this gives a point, where

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\vec{u}) = 0 \tag{3.35}$$

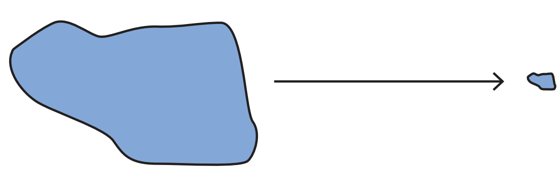


Figure 3.5: Taking $V \to 0$

Use the vector identity $\nabla \cdot (\rho\vec{u}) \equiv \rho(\nabla \cdot \vec{u}) + \vec{u} \cdot \nabla\rho$:

$$\frac{\partial\rho}{\partial t} + \rho(\nabla \cdot \vec{u}) + \vec{u} \cdot \nabla\rho = 0 \tag{3.36}$$

$$\boxed{\frac{D\rho}{dt} + \rho(\nabla \cdot \vec{u}) = 0} \tag{3.37}$$

$$\Rightarrow \frac{D\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{3.38}$$

Note: This means that if there is a variation of density in time and/or space, it must result in dilatation. One consequence is: **If density is constant, then dilatation is zero.**

3.13 Momentum Conservation: Differential Form

By arguments similar to those used with the continuity equation, the integral form of the momentum equation is reduced to the differential form (See textbook for derivation). Writing in terms of components,

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x + F_{x,\text{viscous}} \tag{3.39}$$
$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \rho f_y + F_{y,\text{viscous}} \tag{3.40}$$
$$\rho\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \rho f_z + F_{z,\text{viscous}} \tag{3.41}$$

while f is body force per unit mass, and F is viscous force per unit volume. In vector form,

$$\rho\frac{D\vec{u}}{Dt} = -\nabla p + \rho\vec{f} + \vec{F}_{\text{viscous}} \tag{3.42}$$

3.14 Energy Equation: Differential Form

The energy equation, reduced to differential form, is:

$$\rho\frac{D\left(e + \dfrac{u^2+v^2+w^2}{2}\right)}{Dt} = \rho q - \nabla\cdot(p\vec{u}) + \rho(\vec{f}\cdot\vec{u}) + Q_{\text{viscous}} + W_{\text{viscous}} \tag{3.43}$$

Here, e is internal energy per unit mass, q is heat transfer rate (per unit time) per unit mass, Q is heat transfer per unit volume, f is body force per unit mass, W is work per unit volume. The equation says that the rate of change of energy per unit volume, which consists of internal energy of the molecules and kinetic energy of the flow, is equal to the sum of the heat transfer rate per unit volume, the work done by pressure on the flow (which is negative of work done by the flow against pressure), the work done by body forces and viscous forces, and the rate of heat transfer due to viscous effects, into the flow. We have not included "potential energy" due to a gravitational field or an electromagnetic field in the above. Gravitational potential is important when dealing with water flows, or with balloon flight (aerostatics).

3.15 Simplifying the Momentum Equation

Reynolds number:

$$Re \equiv \frac{\rho U L}{\mu} \quad \text{Ratio of "inertial forces" to "viscous forces"} \tag{3.44}$$

For typical speeds and sizes encountered in flows over aircraft, the Reynolds number is very high (on the order of several millions). This allows us to neglect some terms involving viscous stresses,

compared to other terms in the momentum equation.

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \rho \vec{f} + \vec{F}_{\text{viscous}} \tag{3.45}$$

Neglecting the viscous terms simplifies the momentum equation down to the "Euler equation."

The Euler Equation

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \rho \vec{f} \tag{3.46}$$

Where body forces can be neglected (i.e., except flows with very high radial acceleration)

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p \tag{3.47}$$

3.16 Spinning Cylinder Example

A spinning cylinder is oriented such that the rotation about its axis is $68i + 47j$ radians per second, it is in a freestream 40 km/s, with the fluid density being 1.2 kg/m^3. Find the lift vector.

$$L' = \rho \vec{U}_\infty \times \vec{\Gamma} \tag{3.48}$$

$$\begin{aligned} \rho \vec{U}_\infty \times \vec{\Gamma} &= \rho \begin{vmatrix} i & j & k \\ 68 & 47 & 0 \\ 0 & 0 & 40 \end{vmatrix} && (3.49) \\ &= 40\rho(47i - 68j) && (3.50) \\ &= 2256i - 3264j \;\; \text{Newtons} && (3.51) \end{aligned}$$

Hey, is this perpendicular to the other two vectors? Do the directions make sense? Please verify for yourself.
What is the magnitude of the lift? $|L'| = \sqrt{2256^2 + 3264^2} = ?$

3.17 Bernoulli Equation

Derive the Bernoulli equation for steady incompressible flow, along a streamline.
Euler equations:

$$\begin{aligned} \rho \frac{D\vec{u}}{Dt} &= -\nabla p && (3.52) \\ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} &= \frac{-\nabla p}{\rho} && (3.53) \end{aligned}$$

Steady flow:

$$\frac{\partial}{\partial t}(\,) = 0 \tag{3.54}$$

Vector identity:

$$\begin{aligned} (\vec{u} \cdot \nabla)(\,) &= (u\vec{i} + v\vec{j} + w\vec{k}) \cdot \left(\frac{\partial}{\partial x}(\,)\vec{i} + \frac{\partial}{\partial y}(\,)\vec{j} + \frac{\partial}{\partial z}(\,)\vec{k} \right) && (3.55) \\ (\vec{u} \cdot \nabla)(\vec{u}) &= \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \right)(\vec{u}) && (3.56) \end{aligned}$$

Therefore,

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{3.57}$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{3.58}$$

$$\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{1}{\rho}\frac{\partial p}{\partial z} \tag{3.59}$$

Multiply the first equation by dx, second by dy, and third by dz. Consider the first equation:

$$u\frac{\partial u}{\partial x}dx+v\frac{\partial u}{\partial y}dx+w\frac{\partial u}{\partial z}dx=-\frac{1}{\rho}\frac{\partial p}{\partial x}dx \tag{3.60}$$

Along a streamline, $v/u = dy/dx$ etc. So, $vdx = udy$, etc.

$$u\frac{\partial u}{\partial x}dx+u\frac{\partial u}{\partial y}dy+u\frac{\partial u}{\partial z}dz = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx \tag{3.61}$$

$$u(du) = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx \tag{3.62}$$

Adding the 3 equations together, we get

$$\frac{1}{2}d(u^2+v^2+w^2)=\frac{1}{\rho}(dp) \tag{3.63}$$

$$dp+\frac{\rho}{2}d\left(|\vec{U}|^2\right)=0 \tag{3.64}$$

$$\int_{p_1}^{p_2}dp+\int_{U_1^2}^{U_2^2}\frac{\rho}{2}d\left(|\vec{U}|^2\right)=0 \tag{3.65}$$

$$\boxed{p_2+\frac{\rho}{2}U_2^2=p_1+\frac{\rho}{2}U_1^2=\text{const.}=p_0} \tag{3.66}$$

Chapter 4

Potential Flow and the Laplace Equation

4.1 Objective and Strategy of the Potential Flow Method

Objective:

Get a method to describe flow velocity fields and relate them to surface shapes consistently.

Strategy:

Describe the flow field as the effect of variations of one quantity.

Vector Identity: Given a scalar function Φ

$$\nabla \times (\nabla \Phi) \equiv 0 \tag{4.1}$$

Φ may be $\Phi(x, y, z, t)$.
The "curl" of the gradient of a scalar function is zero.

4.2 Velocity Potential

We defined the "vorticity" of flow as $\xi = \nabla \times \vec{U}$. This is a measure of the rotation of the flow. So $\nabla \times \vec{U} = 0$ means "irrotational flow."

Since velocity is a vector, and is a function of (x, y, z, t), this means that if the flow is irrotational, you can define a scalar function $\Phi(x, y, z, t)$ such that

$$\vec{U} = \nabla \Phi \tag{4.2}$$

$\Phi(x, y, z, t)$ is called the "velocity potential." The definition implies:

$$u = \frac{\partial \Phi}{\partial x} \quad v = \frac{\partial \Phi}{\partial y} \quad w = \frac{\partial \Phi}{\partial z} \tag{4.3}$$

The condition for existence of a velocity potential is that the flow should be IRROTATIONAL. The speed of Mach number need not be low.

4.3 Incompressible Flow

The continuity equation (conservation of mass) in differential form is

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \vec{u}) = 0 \quad \text{or} \quad \frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{4.4}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{4.5}$$

Suppose that the changes in density anywhere in this flow are very small, and hence negligible.

$$\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{4.6}$$

If density is zero, there is no mass, and hence no flow. So the useful solution is

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0} \quad \text{“Dilatation is zero”} \tag{4.7}$$

4.4 Incompressible Potential Flow: Laplace Equation

Incompressible: Continuity Equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4.8}$$

Irrotational:

$$\nabla \times \vec{U} = 0 \tag{4.9}$$

So the velocity can written as the gradient of a scalar Potential

$$u = \frac{\partial \Phi}{\partial x} \qquad v = \frac{\partial \Phi}{\partial y} \qquad w = \frac{\partial \Phi}{\partial z} \tag{4.10}$$

Substituting, the **Continuity equation becomes the Laplace Equation**

$$\boxed{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0} \quad \text{or} \quad \boxed{\nabla^2 \Phi = 0} \tag{4.11}$$

So, what is the Laplace Equation, for incompressible potential flow?
It is simply the continuity equation! Same equation describes STEADY or UNSTEADY incompressible potential flow.

People have found several neat solutions to this equation. They have also found that it is a LINEAR equation, so that you can add one solution to another, and get a third solution! So we get the solution to flow around an airplane, as the sum of, say, 3000 little solutions to the Laplace equation, distributed all over the flow and airplane surface. The difficulty is in calculating the values of these 3000 (or whatever our computer can handle) so that their sum “obeys the boundary conditions” that define our flow problem.

4.5 Laplace Equation Approach

For incompressible flow conditions, velocity is not large enough to cause appreciable density changes, so **density** is known – constant. Thus the unknowns are **velocity** and **pressure**. We need two equations. The Continuity equation is enough to solve for velocity as a function of **time** and **space**.

- Pressure can be obtained from the Bernoulli equation, which comes from Momentum Conservation.
- If the flow is irrotational as well, we can define a potential, and reduce the continuity equation to the form of the Laplace equation.

$$\nabla^2 \Phi = 0 \tag{4.12}$$

$$\vec{U} = \nabla \Phi \qquad u = \frac{\partial \Phi}{\partial x} \qquad v = \frac{\partial \Phi}{\partial y} \qquad w = \frac{\partial \Phi}{\partial z} \tag{4.13}$$

The flow around a complete aircraft is routinely calculated from this.

Problem: If the flow is irrotational, how can there be lift?
Solution: We will introduce rotation, and declare the source of rotation to be "outside" the irrotational flow. This strange logic works, as we will see.

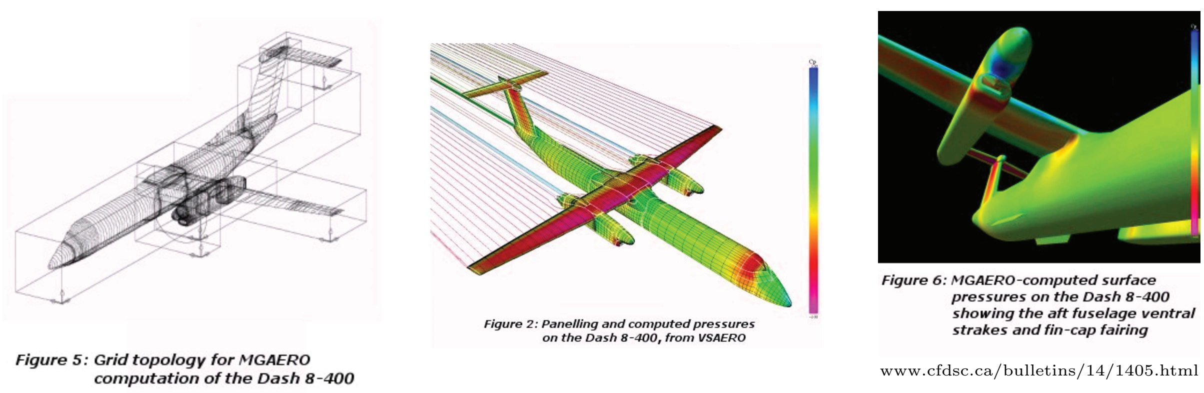


Figure 4.1: Airplane Panel Code

4.6 Building blocks

If each of the following: Φ_1, Φ_2, Φ_3, Φ_4 is a solution of the Laplace equations, then $A\Phi_1 + B\Phi_2 + C\Phi_3 + D\Phi_4$ is also a solution, where A, B, C, D etc. are constants.

Thus, the solution for a complex problem can be expressed as the sum of solutions of several simpler problems. We will use these building blocks to analyze several flows:

1. Uniform flow + source.
2. Uniform flow + source + sink.
3. As source and sink come close together, you get a doublet.
4. Uniform flow + vortex. This looks like flow over a spinning cylinder.

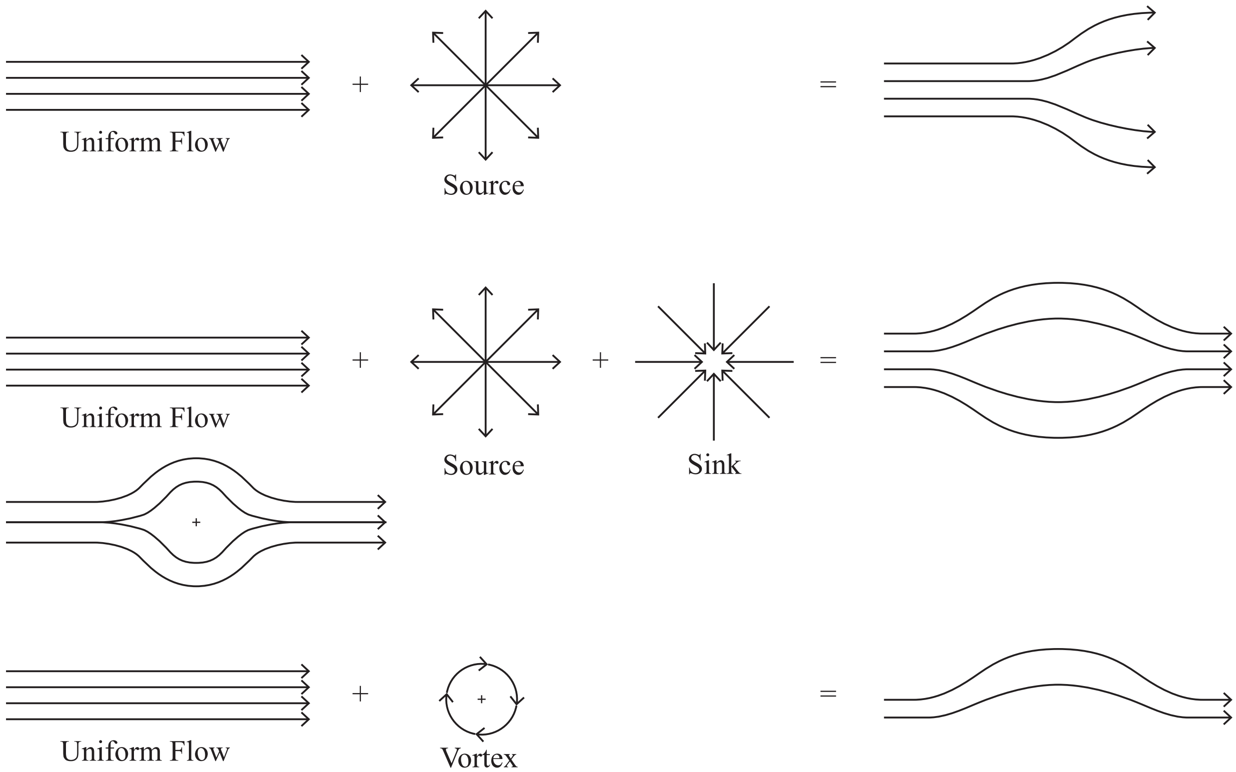


Figure 4.2: Basic solutions

We see that 1., 2. and 3. are all symmetric about the freestream direction. 4. is not.

4.7 Boundary Conditions

This is what specifies the details of the problem in precise mathematical terms.

(i) Along the surface of the airfoil, the flow must be tangential to the surface: Component of velocity normal to the surface is zero. i.e.,

$$\nabla\Phi\cdot\vec{n}=0 \qquad \vec{U}\cdot\vec{n}=0 \tag{4.14}$$

$$\text{or} \qquad \frac{\partial\Phi}{\partial n}=0 \tag{4.15}$$

Thus, all the flow near the surface must be tangential to the surface. In other words, the streamline closest to the surface must be parallel to the surface. Thus another way to express condition (i) for 2-d problems is to say that "the surface is a streamline."

$$\left(\frac{v}{u}\right)_{\text{surface}}=\left(\frac{dy}{dx}\right)_{\text{surface}} \tag{4.16}$$

(ii) The disturbance due to an object must die away as you go far from the surface (any solution which says otherwise is physically unrealistic). Thus,

$$\text{As } x,y,z\to\pm\infty,\ \ \vec{U}\to\vec{U}_\infty \text{ or } \left|\vec{U}-\vec{U}_\infty\right|\to 0 \tag{4.17}$$

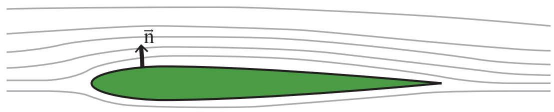


Figure 4.3: Far Field Boundary Condition

Example:
Suppose the general solution to a differential equation is $\phi = Ae^{x} + Be^{-x}$. What can you say about A or B from the above condition?

4.8 Elementary Solutions of the Laplace Equation

(i) Uniform flow:

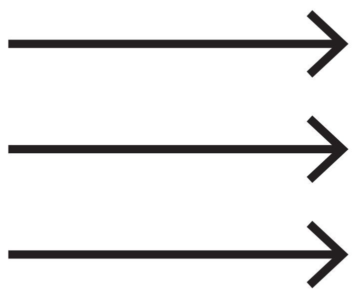


Figure 4.4: Uniform flow

$$u = u_\infty = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{y} = 0 \tag{4.18}$$

$$\phi = u_\infty + f(y) \qquad \phi = \text{const.} + g(x) \tag{4.19}$$

$$\Rightarrow f(y) = \text{const.} \qquad g(x) = u_\infty x \tag{4.20}$$

$$\boxed{\phi = u_\infty x + \text{const.}} \tag{4.21}$$

When we are using only velocity (which is the gradient of the potential), it makes no difference whether you have the constant:

$$\phi = u_\infty x \tag{4.22}$$

(ii) Source or sink (note: a sink is simply a negative source)

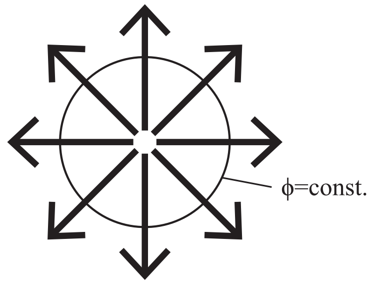


Figure 4.5: Source Potential

At the origin, there is mass flow being added or subtracted, so this is a "singular point" or "singularity." There's flow out of the plane if 2-D. Except at the origin,

$$\nabla \cdot \vec{u} = 0 \tag{4.23}$$

$$u_r = \frac{c}{r} \qquad u_\theta = 0 \tag{4.24}$$

$$c = \frac{\lambda}{2\pi} \tag{4.25}$$

where λ is the source strength [m^2/s].

$$\frac{\partial \phi}{\partial r} = u_r = \frac{\lambda}{2\pi r} \quad \Rightarrow \quad \phi = \frac{\lambda}{2\pi} \ln r + f(\theta) \tag{4.26}$$

$$\frac{1}{r}\frac{\partial \phi}{\partial \theta} = u_\theta = 0 \quad \Rightarrow \quad \phi = \text{const.} + f(r) \tag{4.27}$$

$$\boxed{\phi = \frac{\lambda}{2\pi} \ln r} \tag{4.28}$$

At the origin, there is mass flow being added or subtracted ("singular point"). There's flow out of the plane if 2-D.

χ is a constant, related to the volume flow from/to the source/sink.

(iii) Doublet

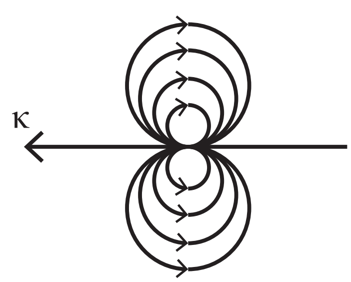


Figure 4.6: Doublet Potential

$$\phi = \frac{\kappa}{2\pi}\frac{\cos\theta}{r} \tag{4.29}$$

(iv) Vortex

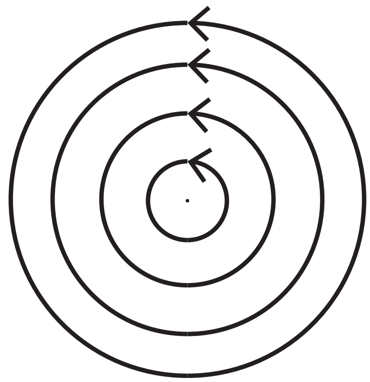


Figure 4.7: Vortex Potential

Let us take the "circulation" around the vortex at radius r.

$$\Gamma = -\oint \vec{u}\cdot dl \tag{4.30}$$

$$\Gamma = -2\pi r u_\theta \tag{4.31}$$

$$u_\theta = -\frac{\Gamma}{2\pi r} \tag{4.32}$$

Note that if you take the circulation at any radius, you'll get the same value of Γ. The only vorticity is concentrated at the center: the flow is irrotational everywhere except at the center.

$$\frac{\partial\Phi}{\partial r} = u_r = 0 \qquad \frac{1}{r}\frac{\partial\Phi}{\partial\theta} = u_\theta = -\frac{\Gamma}{2\pi r} \tag{4.33}$$

$$\boxed{\Phi = -\frac{\Gamma}{2\pi}\theta} \tag{4.34}$$

4.9 Line Vortex

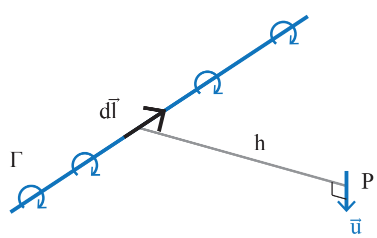


Figure 4.8: Line Vortex

Consider a segment of a vortex filament as shown, with strength Γ. Velocity induced at point P by the segment dl is

$$d\vec{u} = \frac{\Gamma}{4\pi}\frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3} \tag{4.35}$$

When θ is the angle between $d\vec{l}$ and $\vec{r}$,

$$d\vec{l} \times \vec{r} = |d\vec{l}||\vec{r}| \sin\theta \tag{4.36}$$

Thus, velocity induced at P by an infinite, straight vortex filament is

$$u = \frac{\Gamma}{2\pi h} \tag{4.37}$$

where h is the distance $\perp^r$ to the vortex sheet. Thus, the induced velocity drops as h increases.

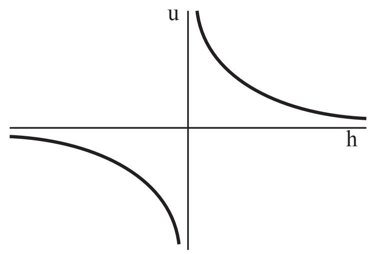


Figure 4.9: Induced Velocity

Note: This is a general result for potential fields, taken by analogy from the result of the magnetic field induced by a segment $d\vec{l}$ of a conductor carrying current I, in a medium of permeability μ:

$$d\vec{B} = \frac{\mu I}{4\pi}\frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3} \tag{4.38}$$

4.10 Vortex Sheet

A vortex sheet is a continuous sheet of vortices. It is used to represent a "shear layer" across which the velocity changes, as shown below.

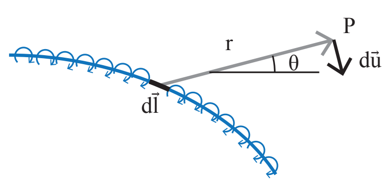


Figure 4.10: Vortex Sheet

$$u = \frac{\Gamma}{4\pi}\int_{-\infty}^{\infty} \frac{\sin\theta}{r^2} dl \tag{4.39}$$

$$\vec{u} = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi}\frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3} \tag{4.40}$$

Chapter 5

Thin Airfoil Theory

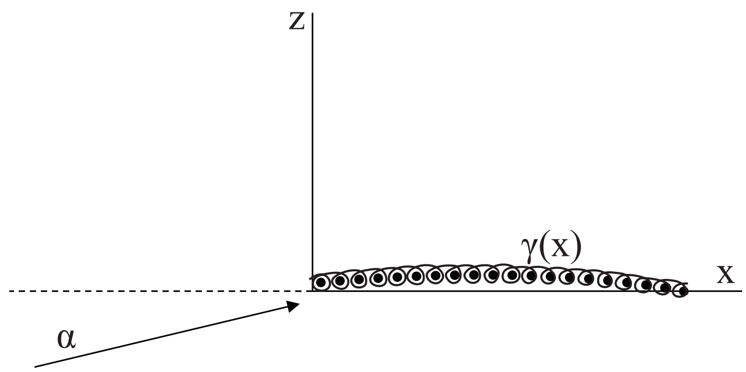


Figure 5.1: Thin Airfoil Theory

The lift and pitching moment produced by a thin cambered airfoil can be represented by a vortex sheet placed along the mean camber line of the airfoil. The issue is to find the unknown variation of vortex sheet strength $\gamma(x)$.

The boundary conditions to satisfy are:

1) Velocity must be tangential to the vortex sheet at the vortex sheet. In other words, the vortex sheet is itself along a streamline.

2) The trailing edge is a stagnation line.

If the airfoil is represented by a vortex sheet along the camber line, then the camber line is a streamline of the flow.

The velocity induced at a point x by the entire vortex sheet is

$$w(x) = -\int_0^c \frac{\gamma(\xi)}{2\pi(x-\xi)} d\xi \tag{5.1}$$

where ξ is the distance along the chord line.

The thin airfoil assumption leads to the argument that $w'(s) \approx w(x)$ where w is the component of velocity normal to the vortex sheet at the camber line.

Boundary Condition:

$$\vec{U}_\infty \cdot \vec{n} + w'(s) = 0 \tag{5.2}$$

$$\vec{U}_\infty \cdot \vec{n} = |\vec{U}_\infty| \left(\alpha - \frac{dz}{dx} \right) \tag{5.3}$$

So, note from the figure:

$$\vec{U}_\infty \cdot \vec{n} = |\vec{U}_\infty| \sin\left(\alpha - \arctan \frac{d}{dx}(z) \right) \approx |\vec{U}_\infty| \left(\alpha - \frac{dz}{dx} \right) \tag{5.4}$$

for small angle of attack and small surface slope.

$$\frac{1}{2\pi} \int_0^c \frac{\gamma(\xi)}{2\pi(x-\xi)} d\xi = |\vec{U}_\infty| \left(\alpha - \frac{dz}{dx} \right) \tag{5.5}$$

$$w(x) = U_\infty \left(\alpha - \frac{dz}{dx} \right) \tag{5.6}$$

$$dw(x) = \frac{\gamma(\xi) d\xi}{2\pi(x-\xi)} \tag{5.7}$$

$$w(x) = \frac{1}{2\pi} \int_0^1 \frac{\gamma(\xi) d\xi}{x-\xi} \tag{5.8}$$

$$\frac{1}{2\pi U_\infty} \int_0^1 \frac{\gamma d\xi}{x-\xi} = \alpha - \frac{dz(x)}{dx} \tag{5.9}$$

$$\cos\theta = 1 - 2x \qquad (\theta \text{ varies from } 0 \text{ to } \pi) \tag{5.10}$$

$$\gamma(\theta) = 2U_\infty \left(A_0 \cot\frac{\theta}{2} + \sum_1^\infty A_n \sin n\theta \right) \tag{5.11}$$

$$[\cos(n-1)x - \cos(n+1)x] = 2\sin x \sin nx \tag{5.12}$$

$$\int_0^\pi \frac{\cos n\theta'}{\cos\theta' - \cos\theta} d\theta' = \pi \frac{\sin n\theta}{\sin\theta} \tag{5.13}$$

$$A_0 - \sum_1^\infty A_n \cos n\theta = \alpha - \frac{dz}{dx} \tag{5.14}$$

$$\boxed{\begin{aligned} A_0 &= \alpha - \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} d\theta && (5.15) \\ A_n &= \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos n\theta d\theta && (5.16) \end{aligned}}$$

If each of the following: Φ_1, Φ_2, Φ_3, Φ_4 is a solution of the Laplace equations, then $A\Phi_1 + B\Phi_2 + C\Phi_3 + D\Phi_4$ is also a solution, where A, B, C, D etc. are constants.

Thus, the solution for a complex problem can be expressed as the sum of solutions of several simpler problems. We will use these building blocks to analyze several flows:

1. Uniform flow + source.

2. Uniform flow + source + sink.

3. As source and sink come close together, you get a doublet.

4. Uniform flow + vortex. This looks like flow over a spinning cylinder.

5.1 Results for a Symmetric Airfoil

Symmetric airfoil: the camber line is straight.

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)}{(x-\xi)}d\xi = U_\infty\alpha \tag{5.17}$$

Use the transformation:

$$\xi = \frac{c}{2}(1-\cos\theta) \tag{5.18}$$

$$x = \frac{c}{2}(1-\cos\theta_0) \tag{5.19}$$

$$d\xi = \frac{c}{2}\sin\theta d\theta \tag{5.20}$$

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\sin\theta}{(\cos\theta_0-\cos\theta_0)}d\theta = U_\infty\alpha \tag{5.21}$$

$$\gamma(\theta) = 2\alpha U_\infty\frac{(1+\cos\theta)}{\sin\theta} \tag{5.22}$$

Consider some features of this function (5.22):
When you go close to the leading edge, the value goes shooting up towards infinity.

Total circulation around the airfoil is:

$$\Gamma = \int_0^c \gamma(\xi)d\xi \tag{5.23}$$

$$\Gamma = \int_0^\pi \gamma(\theta)\sin\theta d\theta \tag{5.24}$$

Therefore,

$$\Gamma = \pi\alpha c U_\infty \tag{5.25}$$

$$L' = \rho U_\infty\Gamma = \rho\pi\alpha c U_\infty^2 \tag{5.26}$$

$$L' = \frac{1}{2}\rho U_\infty^2 c c_l \tag{5.27}$$

$$c_l = 2\pi\alpha \tag{5.28}$$

$$\frac{dc_l}{d\alpha} = 2\pi \tag{5.29}$$

Pitching moment about the leading edge

$$M'_{LE} = -\int_0^c \xi dL = -\rho U_\infty\int_0^c \xi\gamma(\xi)d\xi \tag{5.30}$$

This reduces to

$$M'_{LE} = -q_\infty c^2\frac{\pi\alpha}{2} \tag{5.31}$$

The moment coefficient is

$$c_{mLE} = \frac{M'_{LE}}{q_\infty c^2} = -\frac{\pi\alpha}{2} \tag{5.32}$$

This can also be written as

$$c_{mLE} = -\frac{c_l}{4} \tag{5.33}$$

The center of pressure is at

$$x_{cp} = \frac{-M'_{LE}}{L'} = -\frac{c(c_{mLE})}{c_l} = \frac{c}{4} \tag{5.34}$$

5.2 Cambered Airfoil

Assume that the camber line is given by $z = z(x)$, with $z = 0$ at $x = 0$ and $x = c$. The lift coefficient is:

$$c_l = 2\pi \left[\alpha + \frac{1}{\pi} \int_0^\pi \frac{d}{dx} (\cos\theta_0 - 1) d\theta_0 \right] \tag{5.35}$$

$$\frac{dc_l}{d\alpha} = 2\pi \tag{5.36}$$

Obviously, this consists of two parts: the first part is that due to angle of attack, and second due to camber. When the angle of attack is zero, the lift coefficient is just the second part. The angle of attack for zero lift is thus:

$$\alpha_{l=0} = -\frac{1}{\pi} \int_0^\pi \frac{d}{dx} (\cos\theta_0 - 1) d\theta_0 \tag{5.37}$$

We have to leave these things as integrals because the precise expression will depend on the shape of the camber line for the given airfoil, i.e., the function (dz/dx).

$$c_{mLE} = \frac{-c_l}{4} - \frac{\pi}{4}(A_1 - A_2) \tag{5.38}$$

The coefficients A_1 and A_2 are seen from the following integrals:

$$A_0 = \alpha - \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} d\theta_0 \tag{5.39}$$

$$A_n = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos(n\theta_0) d\theta_0 \tag{5.40}$$

Lift and moment coefficients are:

$$c_{p_l} - c_{p_u} = \frac{\rho U_\infty \gamma}{\rho U_\infty^2/2} = \frac{2\gamma}{U_\infty} = 4 \left(A_0 \cot\frac{\theta}{2} + \sum_1^\infty A_n \sin n\theta \right) \tag{5.41}$$

$$l = \int_0^1 \rho U_\infty \gamma dx \qquad \text{or} \qquad c_l = 2\pi \left(A_0 + \frac{A_1}{2} \right) \tag{5.42}$$

$$m_{l.e.} = \int_0^1 \rho U_\infty \gamma x dx \qquad \text{or} \qquad c_m = -\frac{\pi}{2} \left(A_0 + A_1 - \frac{A_2}{2} \right) = -\frac{c_l}{4} - \frac{\pi}{4}(A_1 - A_2) \tag{5.43}$$

$$c_{m\frac{c}{4}} = -\frac{\pi}{4}(A_1 - A_2) \tag{5.44}$$

The location of the center of pressureis now:

$$x_{cp} = \frac{c}{4} \left[1 + \frac{\pi}{c_l}(A_1 - A_2) \right] \tag{5.45}$$

Chapter 6

Finite Wings

6.1 Helmholtz Vortex Theorems

The starting vortex is a time-rate-of-change issue. The same arguments lead to what are known as the Helmholtz vortex theorems for steady flow.

1. The strength of a vortex filament is constant along its length. This leads to the next argument:
2. A vortex filament cannot begin or end in an irrotational flow; it must extend to the boundaries of the fluid element, or form a closed loop.

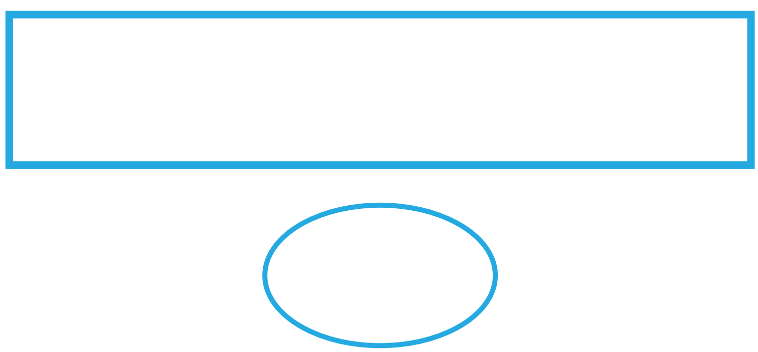


Figure 6.1: Helmholtz Vortex Theorems

From the above, we can infer the vortex system generated by a finite wing moving in an otherwise irrotational flow. Any change in bound circulation along the wing span must be accompanied by the trailing of a vortex filament whose strength is equal and opposite to the said change.

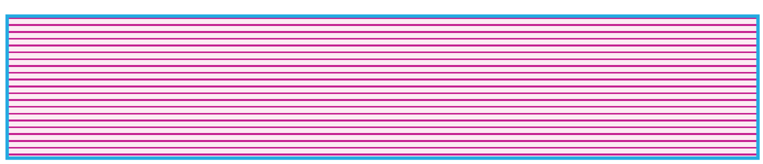


Figure 6.2: Trailing Vortex Sheet

Thus, if the change in bound circulation is continuous, a "sheet" of vorticity, rather than discrete vortices, will be trailed all along the span.

The strength of this trailing vortex sheet at any spanwise station y_0 is thus:

$$\boxed{\gamma(y_0) = -\left(\frac{d\Gamma}{dy}\right)_{y_0}} \tag{6.1}$$

The strength of each element of the trailing vortex sheet in this model, cannot change with x (distance downstream) unless it reaches some other flow boundary.

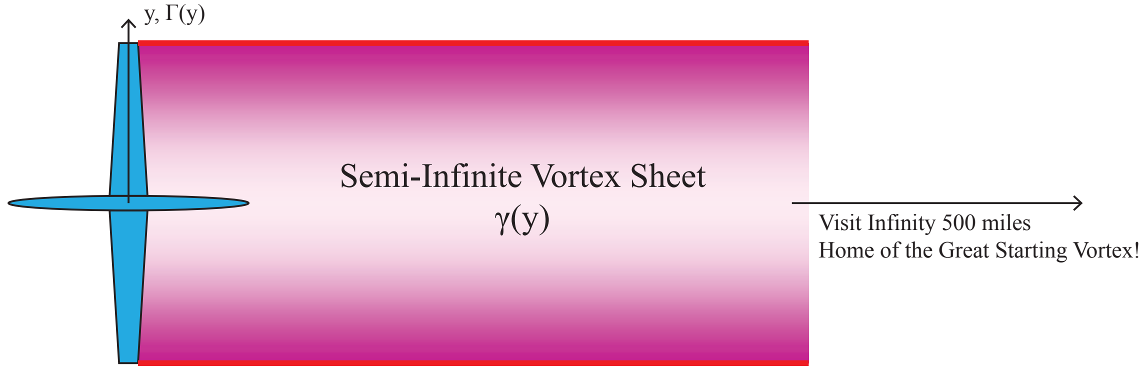


Figure 6.3: Semi Infinite Vortex Sheet

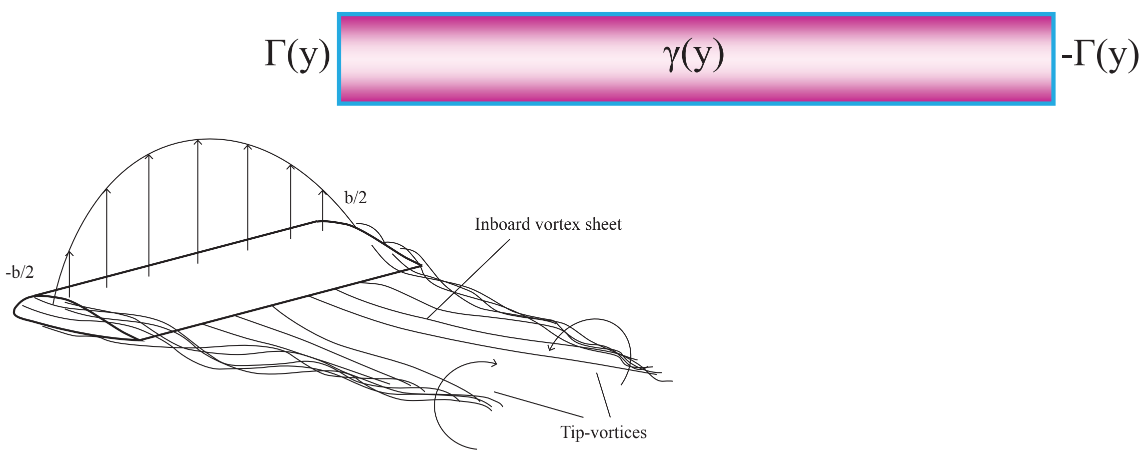


Figure 6.4: Vortex System of a Finite Wing

The vortex system must extend downstream to wherever the bound circulation was first set up: starting vortices must have been created at the same time, and left there forever. This completes the vortex system of the finite wing:

1. Bound circulation, varying along the span, and dropping to zero at each wingtip.
2. Inboard trailing vortex sheet.
3. Tip vortices, one from each wing tip, getting stronger downstream until all of the inboard vortex sheet has rolled up into the vortex on each side.
4. Starting vortex, left behind where the bound circulation was first set up.

In reality, the vortex sheets "rolls up" into the tip vortex as seen above.

6.2 Prandtl's Lifting Line Model

Consists of the following:

a) A straight bound vortex $\Gamma(y)$, placed along the quarter-chord line of the wing.

b) A trailing vortex distribution of strength $\gamma(y)$ such that

$$\boxed{\gamma(y_0) = -\left(\frac{d\Gamma}{dy}\right)_{y_0}} \tag{6.2}$$

The surface boundary condition is that the normal velocity due to the wing + wake + freestream must be zero at the wing surface.

In the lifting line theory, this becomes simple because:

a) The velocity induced by the bound vortex at the bound vortex center line is zero.

b) The wake consists of straight tip vortex + straight vortex filaments trailing back to infinity: semi-infinite line vortices + semi-infinite vortex sheet.

Basis of the theory: The vortex system "induces" downward velocity everywhere inside.

The downward velocity induced at the lifting line (wing) decreases the effective angle of attack seen by the wing, and hence, the lift coefficient is reduced. Also, the new "effective freestream velocity" seen by the wing is at an angle (the induced angle of attack) to the original freestream velocity. Thus the force vector is inclined backwards, and this now has a component along the drag direction. This is "induced drag."

So the problem reduces to finding the induced angle of attack, given the wing geometry, airfoil section characteristics, the freestream velocity, and the wing's "geometric" angle of attack.
In other words,
Effective angle of attack = geometric angle of attack + induced angle of attack, where the induced angle of attack is negative.

$$\alpha_e = \alpha_g + \alpha_i \tag{6.3}$$

Example: Geometric angle of attack = 3 deg. Induced = -0.5 deg. Effective = 2.5 deg.

The effective angle of attack is that corresponding to the bound circulation strength. So if we can find the effective angle of attack, we can find the bound circulation, and hence the lift on the wing. In the following, everything is written as a function of (y), the spanwise station along the wing. This is because the wing may be tapered (chord changes with y), twisted (angle of attack) or even different airfoils may be used inboard and outboard (the zero-lift angle of attack and the lift curve slope of the airfoil section) and anyway, the induced velocity changes with y.

$$c_l(y) = \frac{dC_l}{d\alpha}\left(\alpha_e(y) - \alpha_0(y)\right) \tag{6.4}$$

$$L'(y) = \rho U_\infty \Gamma(y) \tag{6.5}$$

$$L' = \frac{1}{2}\rho U_\infty^2 c(y) c_l(y) \tag{6.6}$$

$$\boxed{\alpha_e = \frac{2\Gamma(y)}{U_\infty c(y)\dfrac{dc_l}{d\alpha}(y)} + \alpha_0(y)} \tag{6.7}$$

The geometric angle of attack is the angle at which one sets the section including the wing twist if any. The induced angle of attack is the angle formed as inverse tangent of the ratio of induced velocity to freestream velocity.

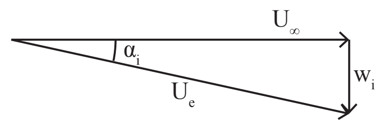


Figure 6.5: Induced Angle of Attack

$$\boxed{\alpha_i = \tan^{-1}\left(\frac{w_i}{U_\infty}\right)} \tag{6.8}$$

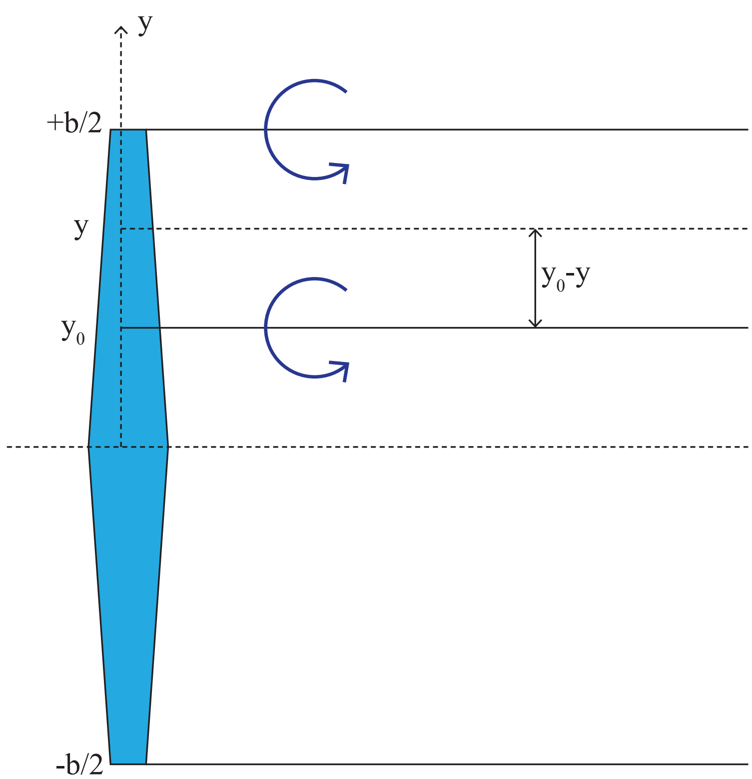


Figure 6.6: Spanwise Variation of Downwash

$$w_i(y) = \int_{-b/2}^{b/2} \frac{\gamma dy}{4\pi(y_0 - y)} \tag{6.9}$$

$$= \int_{-b/2}^{b/2} \frac{-\dfrac{d\Gamma}{dy}dy}{4\pi(y_0 - y)} \tag{6.10}$$

Note on signs:
Γ is positive if the wing has positive lift. $\gamma = d\Gamma/dy$ is negative on the side where y is positive. If y_0 is outboard of y, $y_0 - y$ is positive. The induced velocity w_i at y due to the γ around y_0 is downward, i.e., negative. For the point y_0 shown, $y_0 - y$ is negative. The velocity w_i at y due to the γ around y_0 is positive.

So now our "Fundamental Equation of Lifting Line Theory"

$$\alpha_e = \alpha_g + \alpha_i \tag{6.11}$$

can be expanded using

$$\alpha_i = \tan^{-1}\left(\frac{w_i}{U_\infty}\right) \approx \frac{w_i}{U_\infty} \tag{6.12}$$

$$w_i(y) = \int_{-b/2}^{b/2} \frac{-\dfrac{d\Gamma}{dy}dy}{4\pi(y_0 - y)} \tag{6.13}$$

$$\boxed{\frac{2\Gamma(y)}{U_\infty c(y)\dfrac{dc_l}{d\alpha}(y)} + \alpha_0(y) = \alpha_g(y) + \frac{1}{4\pi U_\infty}\int_{-b/2}^{b/2} \frac{-\dfrac{d\Gamma}{dy}dy}{(y_0 - y)}} \tag{6.14}$$

"Fundamental Equation of Lifting Line Theory"

$$\boxed{\alpha_0(y) - \alpha_g(y) = \frac{1}{4\pi U_\infty}\int_{-b/2}^{b/2} \frac{-\dfrac{d\Gamma}{dy}dy}{(y_0 - y)} - \frac{2\Gamma(y)}{U_\infty c(y)\dfrac{dc_l}{d\alpha}(y)}} \tag{6.15}$$

What is unknown? Only the spanwise Bound Circulation distribution $\Gamma(y)$. Once we have that, we can find $L'(y)$, and integrate that to get L, the lift.

$$L'(y) = L_{\text{effective}}(y)\cos\alpha_i \tag{6.16}$$

$$D_i'(y) = L_{\text{effective}}(y)\sin\alpha_i \tag{6.17}$$

$$\frac{L'}{D_i'}(y) = \frac{1}{\tan\alpha_i} \approx \frac{1}{\alpha_i} \tag{6.18}$$

$$D_i'(y) \approx L'\alpha_i \tag{6.19}$$

6.3 Glauert Solution to the Lifting Line Equation

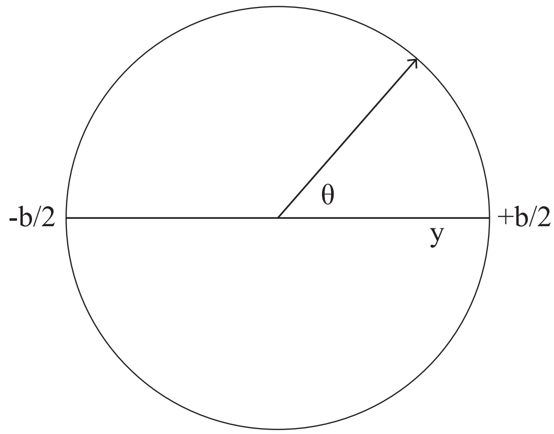


Figure 6.7: Glauert Solution Coordinates

Change coordinates from y to θ.

$$y = \frac{b}{2}\cos\theta \tag{6.20}$$

Assume that the unknown distribution is given by the series

$$\Gamma(\theta) = 2bU_\infty \left[\sum_{n=1}^{N} A_n \sin(n\theta) + \sum_{n=1}^{N} B_n \cos(n\theta)\right] \tag{6.21}$$

with the conditions

$$\Gamma(0) = \Gamma(\pi) = 0 \tag{6.22}$$

This works as long as the distribution does not have discontinuities – it takes only a finite number of terms N to represent the distribution. In practice, $N = 16$ is large enough.

$$\alpha_g - \alpha_{L=0} = \frac{2\Gamma}{a_0 U_\infty C} + \frac{1}{4\pi U_\infty}\int_{-b/2}^{b/2} \frac{\left(\frac{d\Gamma}{dy}\right) dy}{y_0 - y} \tag{6.23}$$

$$y = \frac{b}{2}\cos\theta \qquad dy = -\frac{b}{2}\sin\theta d\theta \tag{6.24}$$

$$\frac{d\Gamma}{dy} = \frac{d\Gamma}{d\theta}\frac{d\theta}{dy} = \left(\frac{d\Gamma}{d\theta}\right)\left(\frac{-2}{b\sin\theta}\right) \tag{6.25}$$

$$\Gamma = 2bU_\infty \left(\sum_{n=1}^{N} A_n \sin n\theta + \sum_{n=1}^{N} B_n \cos n\theta\right) \tag{6.26}$$

$\theta = 0$ and $\pi \quad \Rightarrow \quad \Gamma = 0$ (at the tips) $B_n = 0$ for all n. $\quad \Rightarrow \quad \Gamma = 2bU_\infty \sum_{n=1}^{N} A_n \sin n\theta$ Substituting into the lifting-line equation gives

$$\alpha(\theta_0) = \frac{4b}{m_0(\theta_0)C(\theta_0)} \sum_{n=1}^{N} A_n \sin n\theta_0 + \alpha_{L=0}(\theta_0) + \sum_{n=1}^{N} nA_n \frac{\sin n\theta_0}{\sin\theta_0} \tag{6.27}$$

where $m_0 = \dfrac{dc_l}{d\alpha}$ at $\theta = \theta_0$ corresponding to $y = y_0$.
Note: uses the integral $\displaystyle\int_0^\pi \frac{\cos n\theta d\theta}{\cos\theta - \cos\theta_0} \equiv \pi \frac{\sin n\theta_0}{\sin\theta_0}$.

Satisfy the relation for $\alpha(\theta_0)$ at N values of θ_0 along the span: Gives N equations for A_n, $n = 1, 2, \ldots, N$. Solve.

If we choose N points along the wing, each point is associated with one value θ. Writing this equation at each value of θ, we get N simultaneous equations. Solve for the N unknown coefficients A_n. Then

$$\Gamma(\theta) = 2bU_\infty \sum_{n=1}^{N} A_n \sin n\theta \tag{6.28}$$

$$C_L = \frac{2}{U_\infty S} \int_{-b/2}^{b/2} \Gamma(y) dy = \frac{2b^2}{S} \sum_{n=1}^{N} A_n \int_0^\pi \sin n\theta \sin\theta d\theta \tag{6.29}$$

Then

$$C_L = \frac{2}{Q_\infty S} \int_{-b/2}^{b/2} \Gamma(y) dy \tag{6.30}$$

using

$$\int_0^\pi \sin n\theta \sin\theta d\theta = \begin{cases} \dfrac{\pi}{2} & \text{for } n = 1 \\ 0 & \text{for } n \neq 1 \end{cases} \tag{6.31}$$

$$C_L = A_1 \pi \left(\frac{b^2}{S}\right) = A_1 \pi (AR) \tag{6.32}$$

Lift coefficient depends only on the first term in the series! But you won't get the first term right unless there are enough terms in the series to accurately represent the whole spanwise distribution of the bound circulation.

$$C_{D_i} = \frac{2}{U_\infty S} \int_{-b/2}^{b/2} \Gamma(y) \alpha_i(y) dy \tag{6.33}$$

because

$$D_i' = \int_{-b/2}^{b/2} L'(y) \sin\alpha_i(y) dy \tag{6.34}$$

but $\sin\alpha_i \approx \alpha_i$ for small α_i.

$$\begin{aligned} C_{D_i} &= \frac{2}{U_\infty S} \int_{-b/2}^{b/2} \Gamma(y) \alpha_i(y) dy & (6.35) \\ &= \frac{2b^2}{S} \int_0^\pi \left(\sum_{n=1}^{N} A_n \sin n\theta \right) \alpha_i(\theta) \sin\theta d\theta & (6.36) \end{aligned}$$

$$\alpha_i(\theta) = \sum_{n=1}^{N} n A_n \frac{\sin(n\theta)}{\sin\theta} \tag{6.37}$$

$$\int_0^\pi \sin n\theta \sin\theta d\theta = \begin{cases} \dfrac{\pi}{2} & \text{for } n = 1 \\ 0 & \text{for } n \neq 1 \end{cases} \tag{6.38}$$

$$C_{D_i} = \frac{2b^2}{S}\left(\frac{\pi}{2}\right)\left(\sum_{n=1}^{N} nA_n^2\right) \tag{6.39}$$

$$= \pi(AR)A_1^2\left[1+\sum_{n=2}^{N} n\left(\frac{A_n}{A_1}\right)^2\right] \tag{6.40}$$

$$C_{D_i} = \frac{C_L^2}{\pi(AR)}\left[1+\sum_{n=2}^{N} n\left(\frac{A_n}{A_1}\right)^2\right] \tag{6.41}$$

$$= \frac{C_L^2}{\pi(AR)}(1+\delta) = \frac{C_L^2}{\pi(AR)e} \tag{6.42}$$

6.4 Elliptical Lift Distribution

$$\frac{L}{D_i} = \frac{C_L}{C_{D_i}} = \frac{A_1\pi(AR)}{A_1^2\pi(AR)\left[1+\sum_{n=2}^{N} n\left(\frac{A_n}{A_1}\right)^2\right]} \tag{6.43}$$

Max $\frac{L}{D_i} \quad\Rightarrow\quad A_n = 0$ for $n \geq 2$.

$\Rightarrow\quad \Gamma = 2bU_\infty A_1 \sin\theta = 2bU_\infty A_1\sqrt{1-\cos^2\theta} = 2bU_\infty A_1\sqrt{1-\left(\frac{2y}{b}\right)^2}$

$$\Gamma = \Gamma_0\sqrt{1-\left(\frac{2y}{b}\right)^2} \qquad \text{Equation to an ellipse!} \tag{6.44}$$

For this Γ distribution, downwash is

$$W(\theta_0) = \frac{-\Gamma_0}{2b} \;\; \text{; constant along the span!} \tag{6.45}$$

$$\alpha_i = \frac{-W}{U_\infty} \triangleright \cong \frac{C_L}{\pi(AR)} \tag{6.46}$$

$$C_{D_i} = \frac{C_L^2}{\pi(AR)} \qquad (e=1) \tag{6.47}$$

Another simplification is possible if the distribution is symmetric: no yaw or roll, and you're not carrying a spare engine on one wing, for instance.

$$\Gamma(\theta) = \Gamma(\pi-\theta) \tag{6.48}$$

$A_n = 0$ for even values of n. So,

$$\Gamma = 2bU_\infty \sum_{n=1}^{N} A_{2n-1}\sin(2n-1)\theta \tag{6.49}$$

Thus with just 4 points, you can get a series with 7 terms – up to A_7.

Chapter 7

Viscous Flow

7.1 Viscous Shear Stress

$$\sigma_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \tag{7.1}$$

$$\sigma_{yz} = \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \tag{7.2}$$

$$\sigma_{zx} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \tag{7.3}$$

The direction is indicated by the second subscript. The units of stress are Force/Area, such as N/m^2, psi, or psf.

These can be written in compact form. Here u is used to represent any of the velocity components, and x is used to represent any of the spatial coordinates. i and j representing x,y,z in turn.

$$\sigma_{ij} = \mu \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right) \tag{7.4}$$

Normal Stresses due to Viscosity

$$\sigma_{xx} = 2\mu \frac{\partial u}{\partial x} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \tag{7.5}$$

$$\sigma_{yy} = 2\mu \frac{\partial v}{\partial y} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \tag{7.6}$$

$$\sigma_{zz} = 2\mu \frac{\partial w}{\partial z} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \tag{7.7}$$

Stokes' Hypothesis:

$$\lambda = -\frac{2}{3}\mu \tag{7.8}$$

So the normal stress due to viscosity add up to zero.

7.2 2D Navier-Stokes Equations For Incompressible Flow

Conservation of mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7.9}$$

Conservation of x-momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{7.10}$$

Conservation of y-momentum:

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial y} = \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{7.11}$$

7.3 Exact Solution to the Steady 2D Navier-Stokes Equations For Incompressible Flow

Plane Parallel Steady Flow Between Two Infinite Plates: Couette Flow

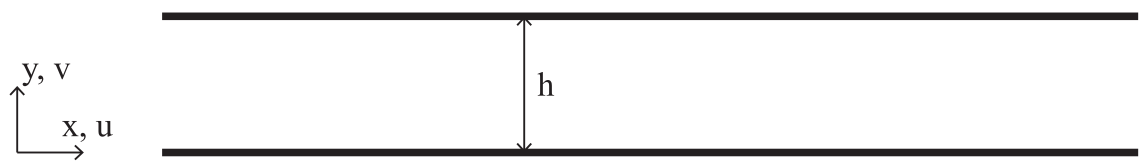


Figure 7.1: Couette Flow

From the problem, there cannot be any change in u along x: So du/dx is zero. Also, $v = 0$. So u is $u(y)$.

$$\frac{1}{\rho}\frac{dp}{dx} = \nu\frac{d^2u}{dy^2} \tag{7.12}$$

$$u(y) = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + Ay + B \tag{7.13}$$

Case 1: Flow between two plates, driven by plate motion

$$u(y) = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + Ay + B \tag{7.14}$$

Boundary conditions: $u = v = 0$ at $y = 0$, $u = U$ and $v = 0$ at $y = h$.

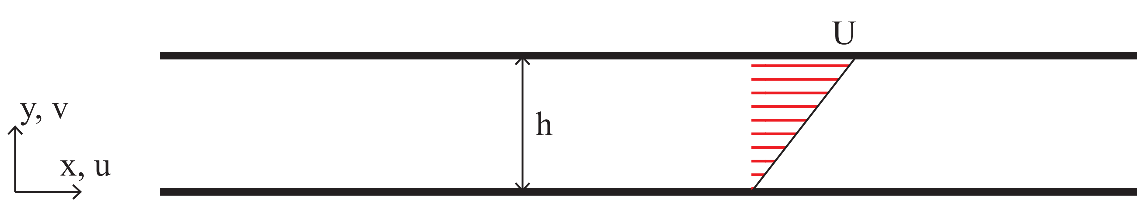


Figure 7.2: Flow Between Driven Plates

$$u(y) = \frac{y}{2\mu}\frac{dp}{dx}(y-h) + \frac{Uy}{h} \tag{7.15}$$

If $dp/dx = 0$,

$$u(y) = \frac{Uy}{h} \qquad \text{Linear velocity profile} \tag{7.16}$$

Shear stress

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\frac{U}{h} \tag{7.17}$$

Skin Friction Coefficient

$$C_f = \frac{\tau_{xy}}{\frac{1}{2}\rho U^2} = \frac{2\mu}{\rho h U} = \frac{2}{Re} \tag{7.18}$$

$$Re = \text{Reynolds number based on } U \text{ and } h = \frac{\rho U h}{\mu} \tag{7.19}$$

Case 2: Flow between two stationary plates, driven by pressure gradient

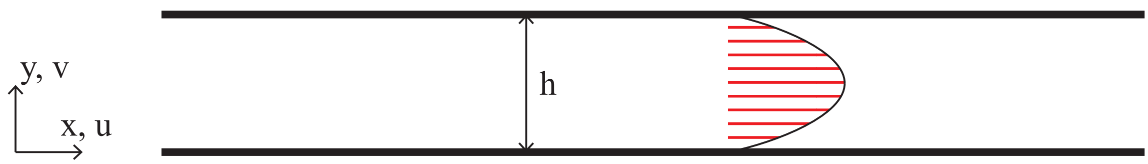


Figure 7.3: Flow Between Stationary Plates

$$u(y) = \frac{y}{2\mu}\frac{dp}{dx}(y-h) \tag{7.20}$$

Note: Sign of dp/dx must be opposite to that of u.

Hagen-Poiseuille Flow: Steady axisymmetric, fully-developed incompressible laminar flow through a constant-diameter pipe

$$\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial (rv)}{\partial r} = 0 \tag{7.21}$$

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial z} = \nu\left[\frac{1}{r}\frac{\partial\left(r\frac{\partial u}{\partial r}\right)}{\partial r} + \frac{\partial^2 u}{\partial z^2}\right] \tag{7.22}$$

$$u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = \nu\left[\frac{1}{r}\frac{\partial\left(r\frac{\partial v}{\partial r}\right)}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right] \tag{7.23}$$

u-momentum equation reduces to

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{\nu}{r}\frac{d\left(r\frac{du}{dr}\right)}{dr} \tag{7.24}$$

Integrating,

$$\frac{du}{dr} = \frac{1}{2\mu}\frac{dp}{dz}r + \frac{C}{r} \tag{7.25}$$

For flow to remain finite at axis $(r = 0)$, $C = 0$. Also, $u = 0$ at the wall, $r = R$.

$$u(r) = -\frac{1}{4\mu}\frac{dp}{dz}(R^2 - r^2) \quad \text{Parabolic Velocity Profile} \tag{7.26}$$

Chapter 8

Boundary Layer

8.1 Boundary Layer Equations

Boundary Layer Approach

$$\begin{array}{c}\text{Newton's 2nd Law}\\ \downarrow \\ \text{Conservation Equations in integral form}\\ \downarrow \\ \text{Convert to differential form}\\ \downarrow \\ \text{Simplify by use of OM analysis}\end{array}$$

Boundary Layer Approximations

At very high Reynolds number ($\sim 100,000+$) based on distance along the surface in the freestream direction,

1. The spatial rate at which properties change across a boundary layer is very large, compared to the rate at which things change along the flow direction. In other words, derivatives with respect to y are much higher than derivatives with respect to x.

$$d(\,)/dy >> d(\,)/dx \tag{8.1}$$

2. Order of magnitude analysis shows that the v-momentum equation reduces to

$$\frac{\partial p}{\partial y} \approx 0 \tag{8.2}$$

   In other words, static pressure is constant across a boundary layer.

u-momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \tag{8.3}$$

For high Reynolds number flows, our order of magnitude analysis says that the u-momentum equation may be approximated as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} \approx \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \tag{8.4}$$

v-momentum equation

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial y}\approx\frac{\mu}{\rho}\nabla^2 v \tag{8.5}$$

becomes

$$\frac{\partial p}{\partial y}\approx 0 \tag{8.6}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8.7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial x}\approx\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \tag{8.8}$$

$$\frac{\partial p}{\partial y}\approx 0 \tag{8.9}$$

a) The variation of pressure across the boundary layer is small. Thus, we can assume that pressure varies only with respect to x. The pressure field may be computed at the edge of the boundary layer from inviscid flow theories, without any knowledge of the boundary layer characteristics.

b) The v-component of velocity within the boundary layer is of the order of $(U_\infty\delta/c)$ where δ is the boundary layer thickness.

c) The boundary layer thickness δ is very small, and for laminar flows it varies inversely as the square root of Reynolds number $\rho U_\infty c/\mu$.

d) We will assume the airfoil surface to be flat, and use a Cartesian coordinate system, neglecting surface curvature. This is because the boundary layer thickness δ is very small compared to the airfoil surface radius of curvature.

8.2 Boundary Layer Features

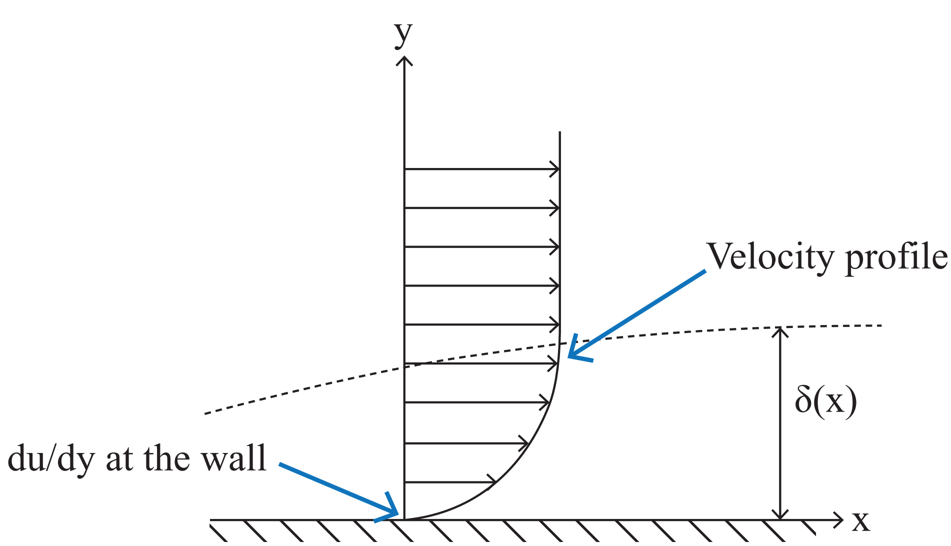


Figure 8.1: Boundary Layer Features

1. Boundary Layer Thickness δ:
   Defined as the $y-$location where u/u_e reaches 0.99%, that is the u-velocity becomes 99% of the edge velocity.

2. Displacement Thickness δ^*:
   This is a measure of the outward displacement of the streamlines from the solid surface as a result of the reduced u-velocity within the boundary layer. This quantity is defined as

$$\delta^* = \int_0^\infty \left[1 - \frac{\rho u}{\rho_e u_e}\right] dy \tag{8.10}$$

3. Momentum Thickness θ:
   This is a measure of the momentum loss within the boundary layer as a result of the reduced velocities within the boundary layer.

$$\theta = \int_0^\infty \frac{\rho u}{\rho_e u_e} \left[1 - \frac{u}{u_e}\right] dy \tag{8.11}$$

4. Shape Factor H:
   This quantity is defined as the ratio δ^*/θ.
   For laminar flows H varies between 2 and 3. It is 3.7 near separation point. Thus excessively large values of H (above 3) indicate that the boundary layer is about to separate. In turbulent flows, H varies between 1.5 and 2.

8.3 Wall Shear Stress and Drag

Surface Shear Stress:
The shear stress at the wall can be found from definition of shear stress

$$\tau_{\text{wall}} = \tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{\text{wall}} \tag{8.12}$$

Skin Friction Coefficient c_f:

$$c_f = \frac{\tau_w}{\left(\frac{1}{2}\rho_e u_e^2\right)} \tag{8.13}$$

Skin Friction Drag D:
Shear stress may be numerically integrated over the entire solid surface to give the skin friction drag force along the x-axis

$$D = \int_{\text{Entire Surface}} \tau_w dx \tag{8.14}$$

Skin Friction Drag Coefficient C_d:
The drag force is usually non-dimensionalized by the freestream dynamic pressure times the chord of the airfoil c, giving the skin friction drag coefficient along the x-axis, C_d.

$$C_d = \frac{D}{\left(\frac{1}{2}\rho_\infty u_\infty^2 c\right)} \tag{8.15}$$

8.4 Thwaites' Integral method for Laminar Incompressible Boundary Layers

This is an empirical method based on the observation that most laminar boundary layers obey the following relationship (Ref: Thwaites, B., Incompressible Aerodynamics, Clarendon Press, Oxford, 1960):

$$\frac{u_e}{\nu}\frac{d(\theta^2)}{dx} = A - B\frac{\theta^2}{\nu}\frac{du_e}{dx} \tag{8.16}$$

Thwaites recommends $A = 0.45$ and $B = 6$ as the best empirical fit.

The above equation may be analytically integrated yielding

$$\theta^2 = \frac{0.45\nu}{u_e^6}\int_0^x u_e^5 dx + \left[\theta^2_{x=0}\frac{u^6_{e x=0}}{u_e^6}\right] \tag{8.17}$$

For blunt bodies such as airfoils, the edge velocity u_e is zero at $x = 0$, the stagnation point. For sharp nosed geometries such as a flat plate, the momentum thickness q is zero at the leading edge. In these cases, the term in the square bracket vanishes.

The integral may be evaluated, at least numerically when u_e is known.

After θ is found, the following relations are used to compute the shape factor H and the shear stress at the wall τ_w. For $0 \leq \lambda \leq 0.1$

$$H = 2.61 - 3.75\lambda + 5.24\lambda^2 \tag{8.18}$$

For $-0.1 \leq \lambda \leq 0$

$$H = 2.472 + \frac{0.0147}{0.107 + \lambda} \tag{8.19}$$

where

$$\lambda = \frac{\theta^2}{\nu}\frac{du_e}{dx} \tag{8.20}$$

$$\tau_{wall} = \frac{\mu u_e}{\theta}(\theta + 0.09)^{0.62} \tag{8.21}$$

Despite the empiricism in the above formulas, Thwaites' integral method is considered to be the best of integral boundary layer methods.

8.5 Transition

Consider the 2-D unsteady, incompressible viscous flow past an airfoil at a sufficiently high Reynolds number

$$Re = \frac{\rho U_\infty c}{\mu} > 300,000 \tag{8.22}$$

We find that the boundary layer over both the upper and lower surfaces may be divided into three regions. They are (1) laminar region, (2) transitional region, and (3) turbulent region.

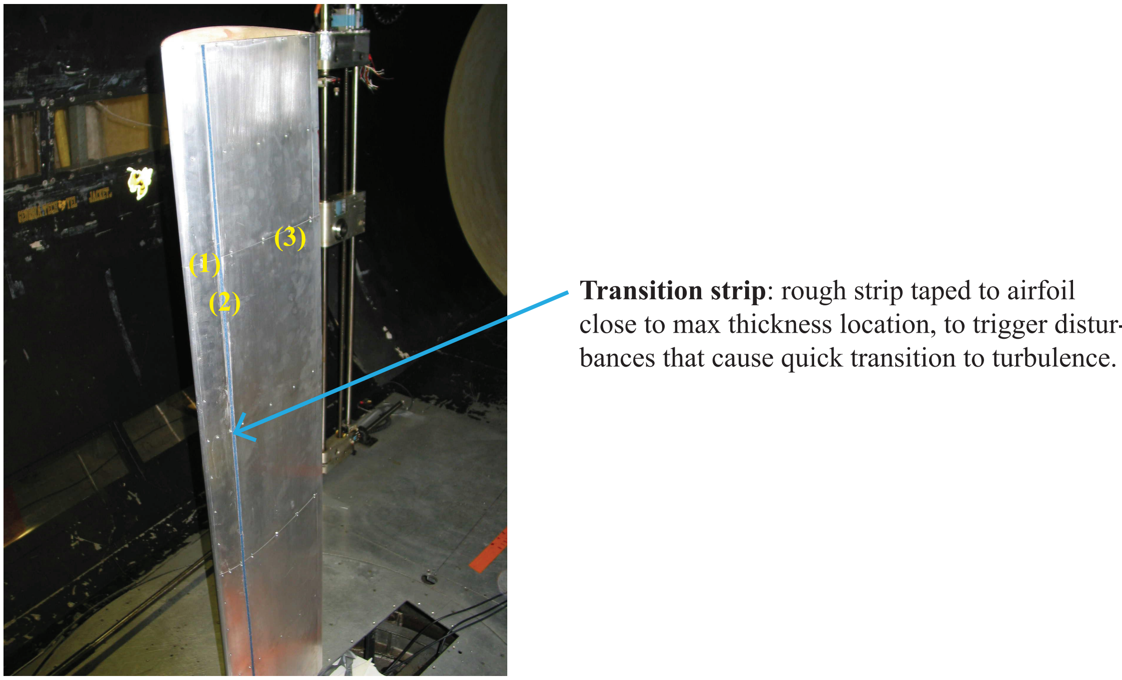


Figure 8.2: Transition

8.6 Turbulence Features

The turbulent region has unsteady flow, although the fluctuations are small and occur about some "steady" mean flow levels. The velocity profile is fuller than in the laminar flow case. Skin friction is higher than in laminar flow.

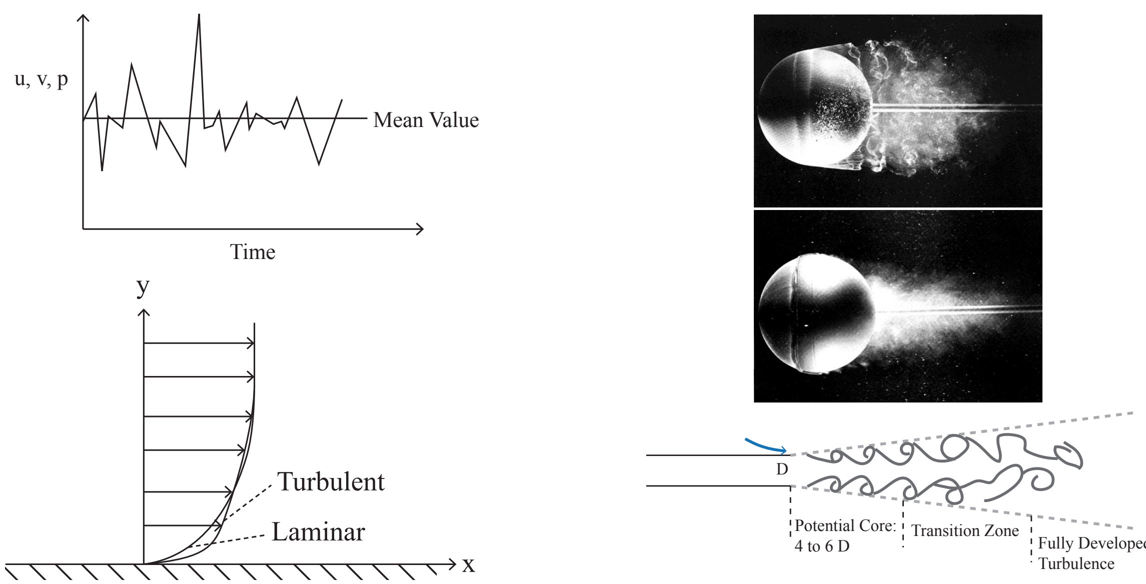


Figure 8.3: Turbulence Features

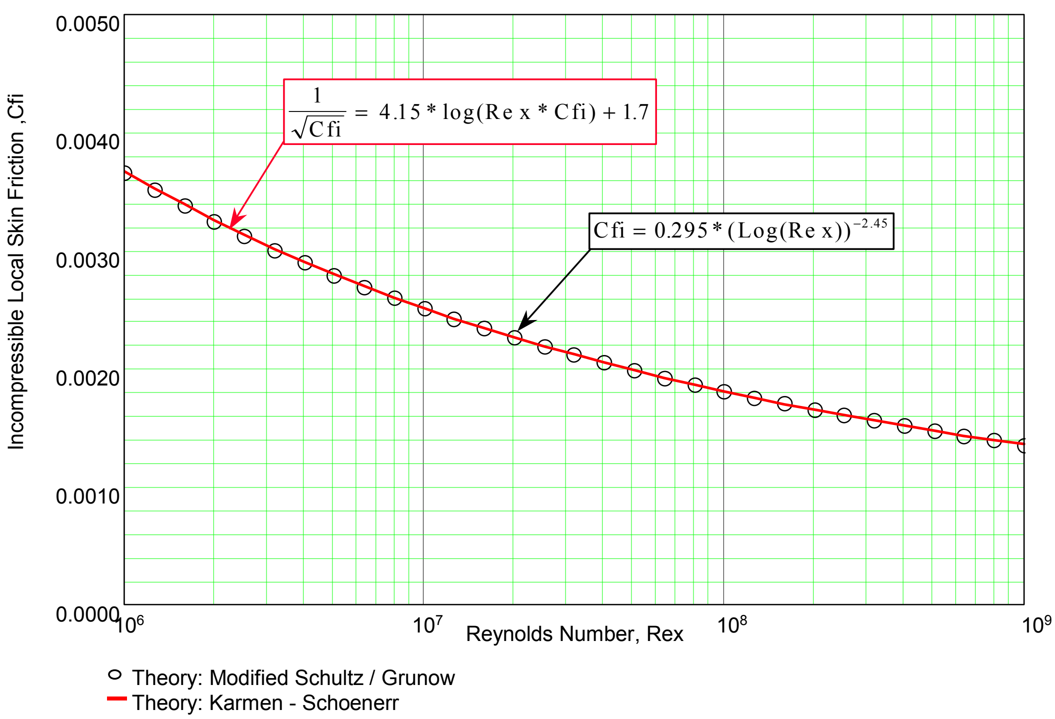
8.7 Incompressible Boundary Layer on Flat Plate at zero angle of attack: Laminar vs. Turbulent

Laminar		Turbulent
$\delta = \dfrac{5.0x}{Re^{0.5}}$		$\delta = \dfrac{0.37x}{Re^{0.2}}$
$\theta = \dfrac{0.664x}{Re^{0.5}}$	$c_f = \dfrac{0.664}{Re^{0.5}}$	
$C_f = \dfrac{1.328}{Re_c^{0.5}}$		$C_f = \dfrac{0.074}{Re_c^{0.2}}$

8.8 Incompressible Turbulent Skin Friction Correlation At Large Reynolds Numbers

Schultz-Grunow equation, modified by Kulfan, Boeing Co.

$$C_{fi} = 0.295(\log Re_x)^{-2.45} \quad ; \quad 10^6 < Re_x < 10^9 \tag{8.23}$$



Courtesy, Dr. R. Kulfan, Boeing Co.

Figure 8.4: Comparison of Incompressible Local Skin Friction Predictions

Chapter 9

Some Practical Results

9.1 Airfoil Drag vs. Angle of Attack

Note: When viscous drag and/or flow separation are present, airfoils do have drag that changes with angle of attack. In other words, C_{D0} of an aircraft is not completely independent of angle of attack, over a large range of angle of attack. This has nothing to do with tip vortices etc. So when we must use an averaged value for C_{D0} when we use lifting line theory and say that for a whole aircraft (note nomenclature).

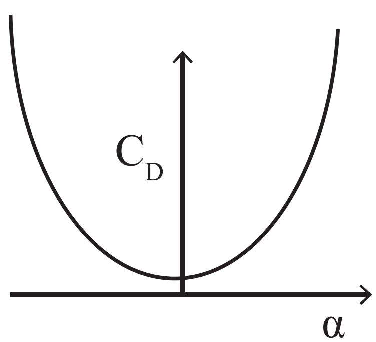


Figure 9.1: Airfoil Drag Coefficient at Angle of Attack

$$C_D = C_{D0} + \frac{C_L^2}{\pi(AR)e} \tag{9.1}$$

The more usual and general way, is to represent the drag as

$$C_D = C_{D0} + KC_L^2 \tag{9.2}$$

K will now be $\dfrac{1}{\pi(AR)e}$.

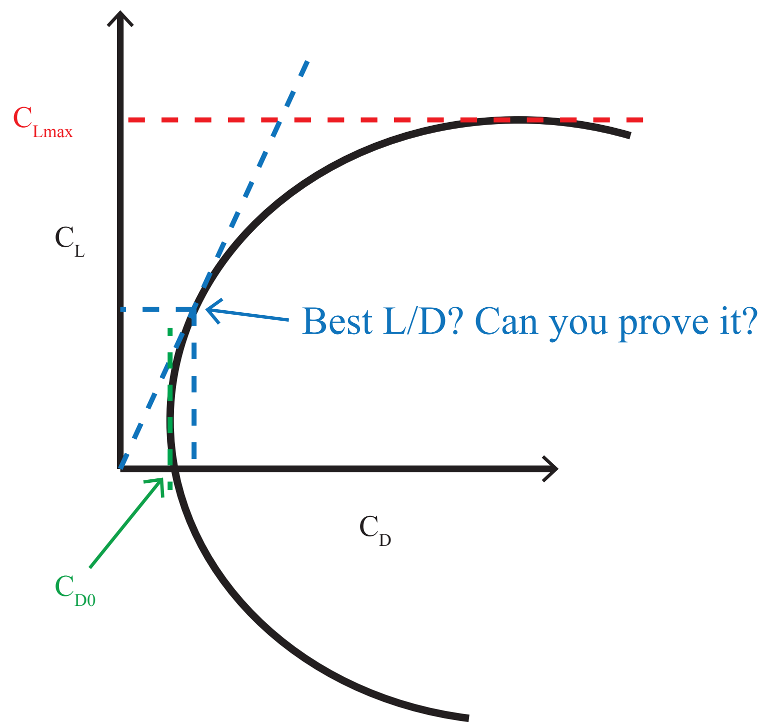


Figure 9.2: Aircraft Drag Polar

9.2 Relation between Boundary Layer Profile and Skin Friction Drag

A positive pressure gradient dp/dx would cause the boundary layer velocity profile to become "unhealthy" – develop an inflection point.

Beyond that, 3 things can happen:

1. Laminar separation
2. Transition to turbulence
3. Turbulent separation

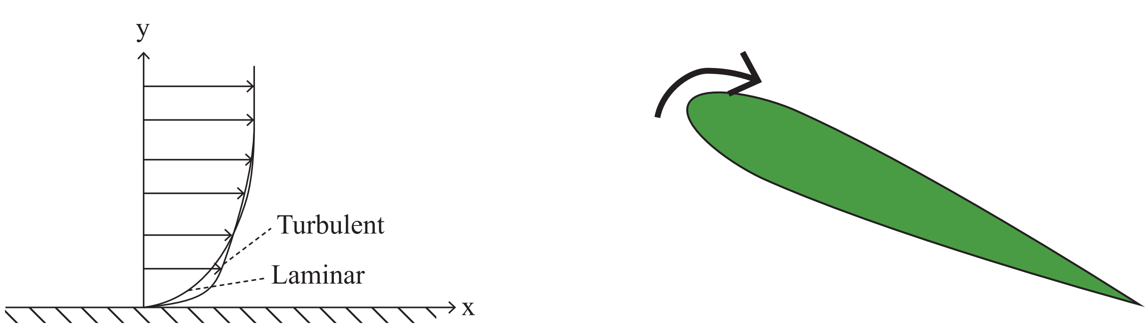


Figure 9.3: Positive pressure gradient would make boundary layer velocity profile "unhealthy"

$$\tau_{\text{wall}} = \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{\text{wall}} \tag{9.3}$$

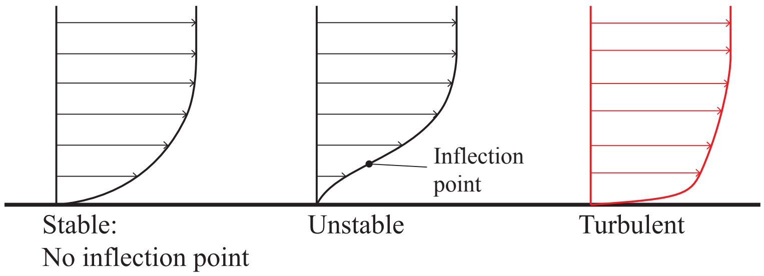


Figure 9.4: Transition to Turbulence

Turbulent boundary layer has high velocities close to surface, large du/dy – high skin friction drag (bad). However, very stable profile delays separation (good!).

9.3 Separation

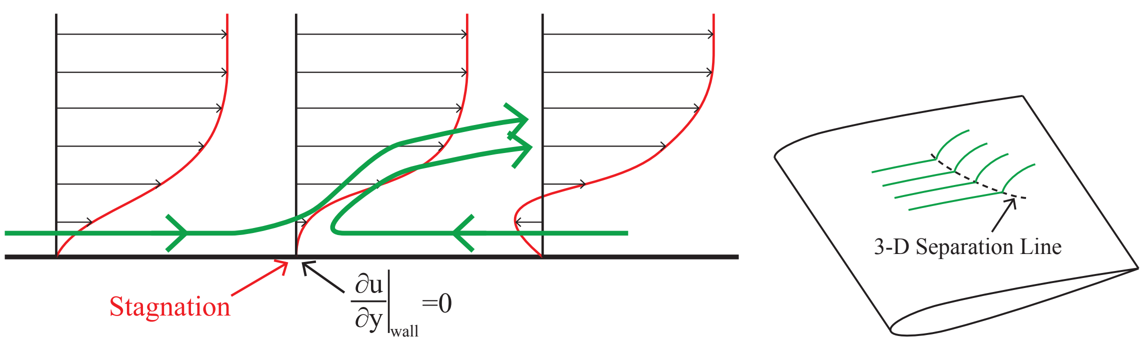


Figure 9.5: Flow Separation

9.4 Separation Control

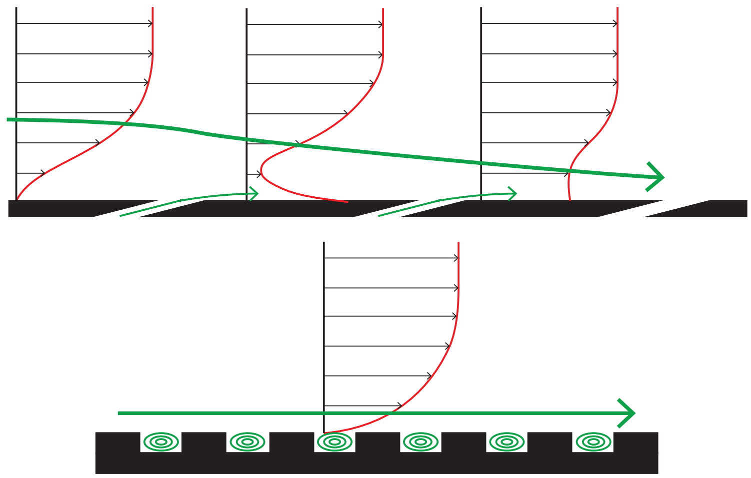


Figure 9.6: Separation Control: surface blowing (upper) and cavities (lower)

9.5 Moving Wall

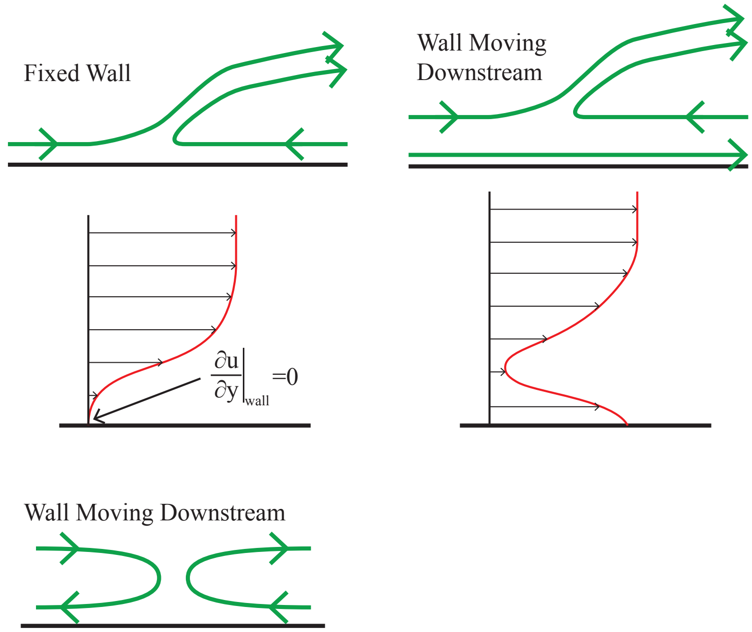


Figure 9.7: Moving Wall Effect

9.6 Boundary Layer Suction

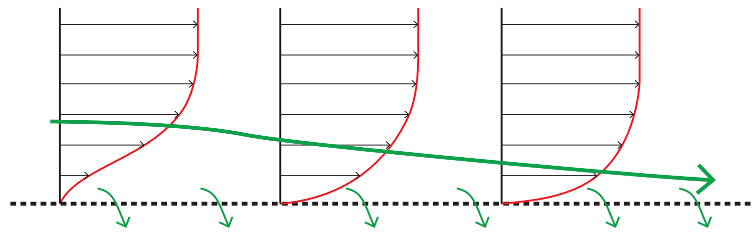


Figure 9.8: Boundary Layer Suction

9.7 Vortex Interactions & Ground Effect

Technique: Think of the velocity induced at the axis of each vortex by every other vortex. A solid wall is treated as an "image plane" with an imaginary mirror image of the vortex on the other side.

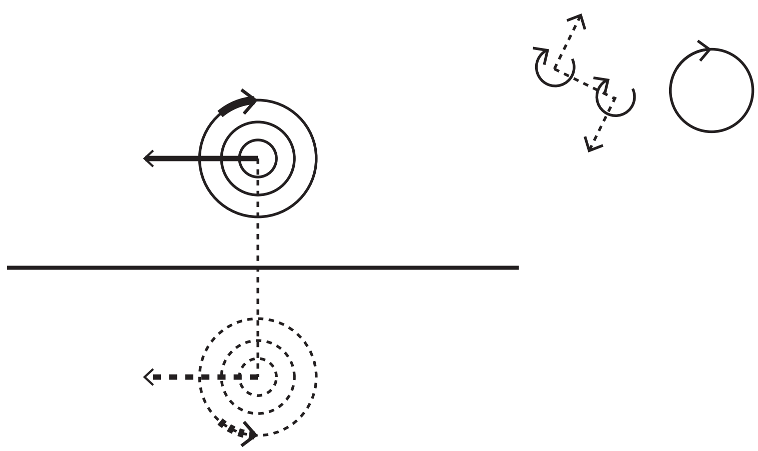


Figure 9.9: Vortex Interactions and Ground Effect

9.8 Potential Flow Panel Code with Interacting Boundary Layer Calculation

"XFOIL" by Prof. M. Drela http://web.mit.edu/drela/Public/web/xfoil/

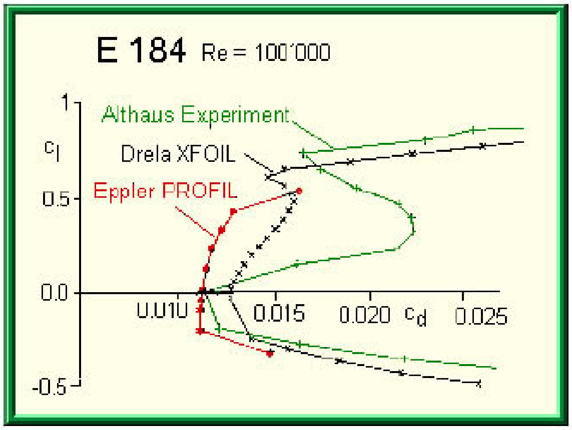


Figure 9.10: Panel Code With Boundary Layer

9.9 Drag Coefficients

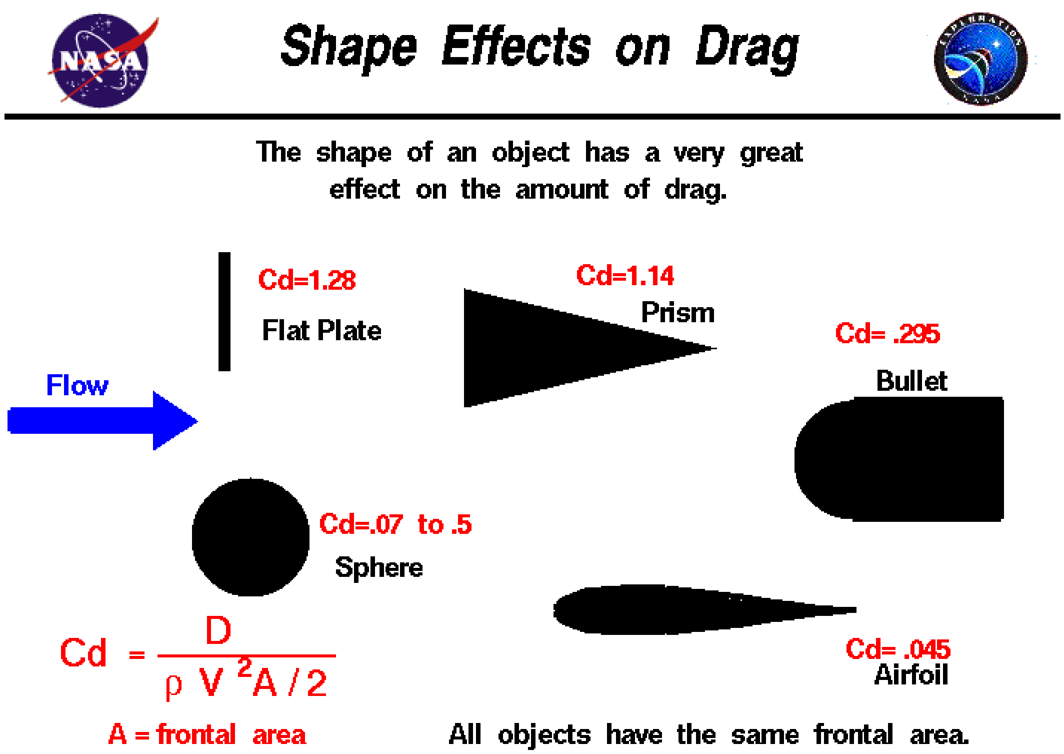


Figure 9.11: Shape Effects on Drag

Table 9.1: Drag Coefficients for bluff bodies

Rough sphere ($Re = 10^6$)	0.40
Smooth sphere ($Re = 10^6$)	0.10
Hollow semi-sphere opposite stream	1.42
Hollow semi-sphere facing stream	0.38
Hollow semi-cylinder opposite stream	1.20
Hollow semi-cylinder facing stream	2.30
Squared flat plate at 90°	1.17
Long flat plate at 90°	1.98
Open wheel, rotating, $h/D = 0.28$	0.58

Table 9.2: Drag Coefficients for streamlined bodies

Laminar flat plate ($Re = 10^6$)	0.001
Turbulent flat plate ($Re = 10^6$)	0.005
Airfoil section, minimum	0.006
Airfoil section, at stall	0.025
2-element airfoil	0.025
4-element airfoil	0.05
Subsonic aircraft wing, minimum	0.05
Subsonic aircraft wing, at stall	0.16
Subsonic aircraft wing, minimum	0.005
Subsonic aircraft wing, at stall	0.09
Aircraft wing (supersonic)	n.a.

Table 9.3: Vehicle Drag Coefficients

Subsonic transport aircraft	0.012
Supersonic fighter, $M = 2.5$	0.016
Airship	0.020-0.025
Helicopter download	0.4-1.2
Sports car	0.3-0.4
Economy car	0.4-0.5
Pickup truck	0.5
Tractor-trailer, with fairings	0.6-0.7
Tractor-trailer	0.7-0.9
Trailer alone	0.9
Racing car	0.65-1.10

The most famous source on drag data is:
"Fluid-Dynamic Drag: Practical Information on Aerodynamic Drag and Hydrodynamic Resistance"
by Sighard F. Hoerner, Dr.-Ing.-habil.